Chapter 2

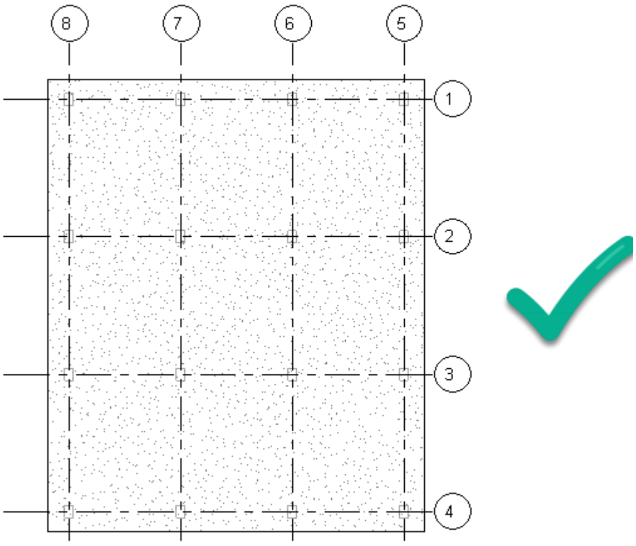
Direct Design Method

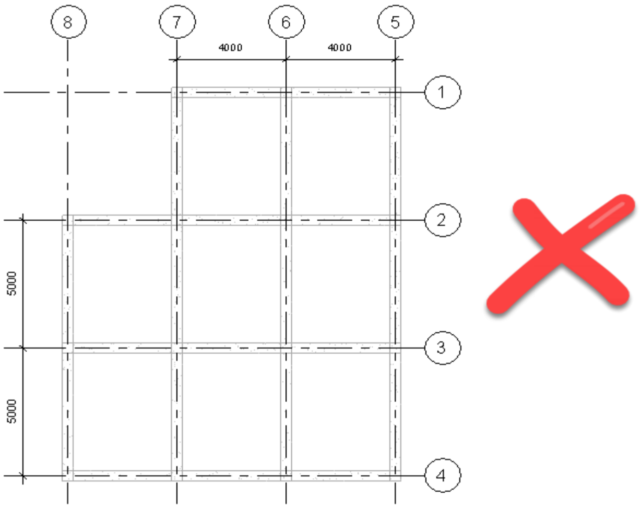
Direct Design Method

The direct design method **(DDM) ACI Code 8.10** is an approximate procedure for the analysis and design of two-way slabs. It is limited to slab systems subjected to uniformly distribution loads and supported on equally or nearly spaced columns. The method uses a set of coefficients to determine the design moments at critical sections. Two-way slab systems that do not meet the limitations of the **ACI Code 8.10.2** must be analysis by more accurate procedures (like equivalent frame or Finite Elements methods).

<u>Limitations for use of direct design method ACI Code 8.10.2</u>

1. There must be a **minimum of three continues** spans in each direction.



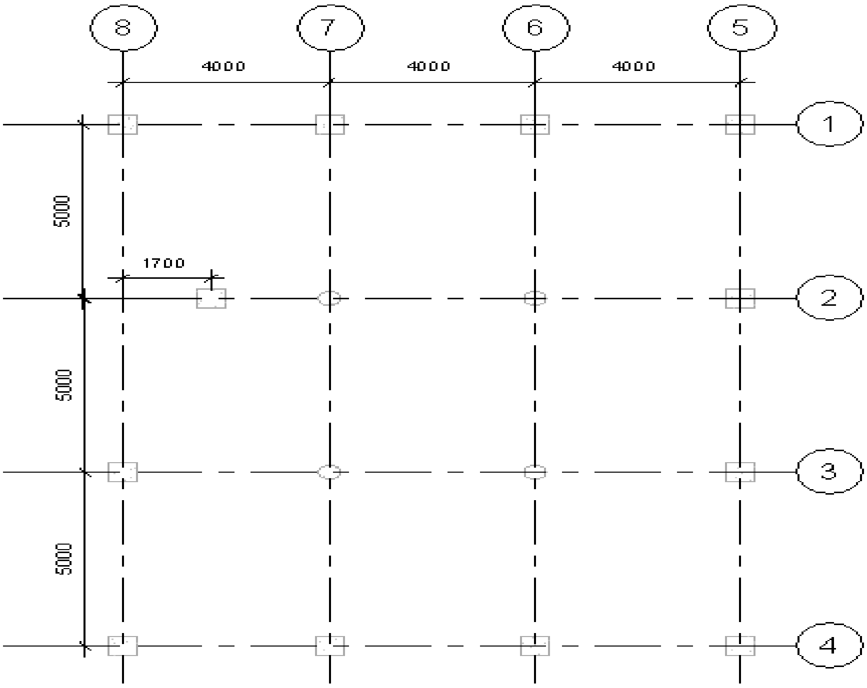


- Thus. A **nine panel** structure **<u>(3 by 3)</u>** is the smallest that can be considered.

2. The panel must be rectangular with the ratio of **the longer to the shorter** spans within a panel **not greater than 2**.

   - If the ratio of the two spans (long span/short span) of a panel **exceeds 2**, the slab resists the moment in the shorter span essentially as **a one-way slab**.

3. Column offset **shall not exceed 10 percent** of the span in direction of offset from either axis between center-lines of successive columns

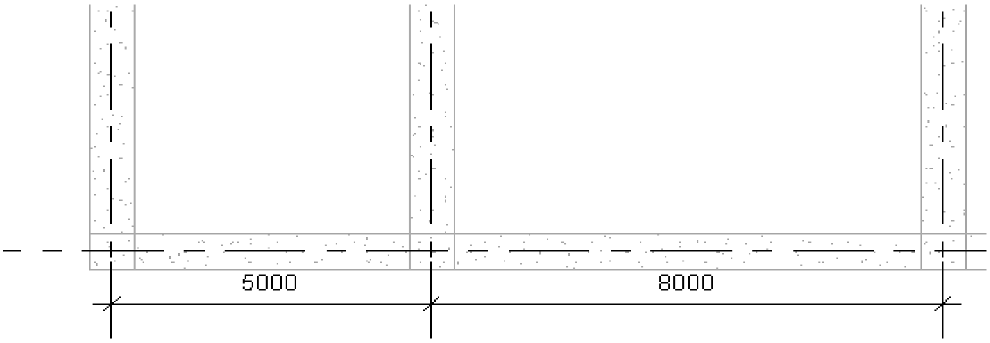


Offset ≤ 0.1ℓ

1700? 0.1×4000

1700 > 400 not O.K

4. Successive span lengths measured center-to-center of supports in each direction shall not differ by more than **one-third the longer span.**

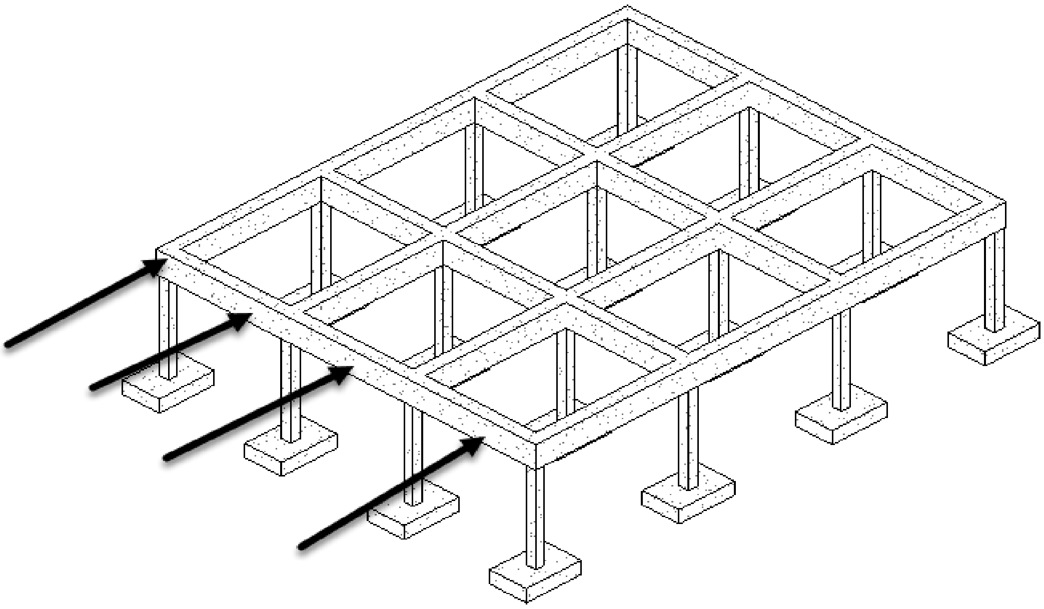


$L_\ell - Ls \leq \frac{1}{3} \times L_\ell$

$8 - 5 ? \frac{1}{3} \times 8$

3 m > 2.67 m not O.K.

5. All loads shall be due to **gravity only** (no lateral loads) and uniformly distributed over an entire panel. The DDM cannot be used for unbraced, laterally loads frames, foundation mats and prestress slabs.



6. Unfactored live load shall not exceed two times the unfactored dead load L.L ≤ 2D.L.

7. **If** beams are used between supports **on all sides**, the relative stiffness ratio of the beams in the two perpendicular directions must be between 0.2 and 0.5.

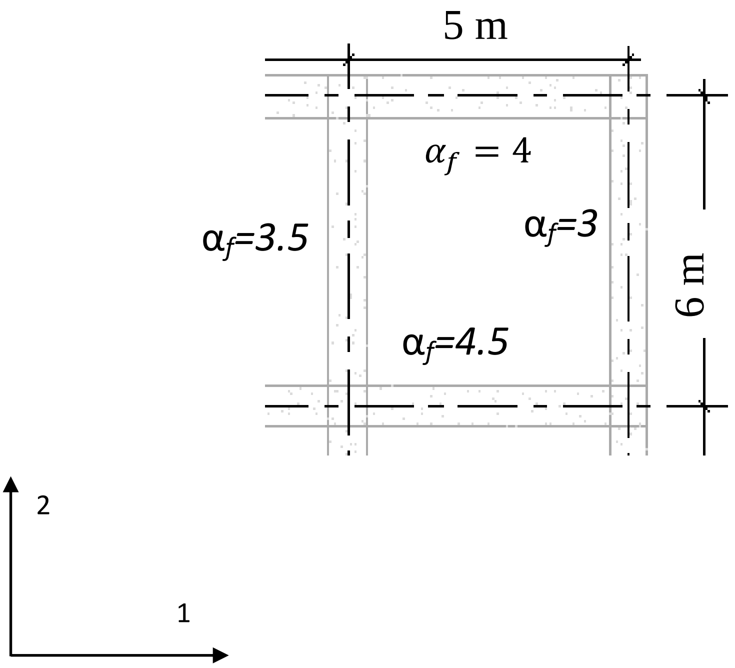
$$0.2 \le \frac{\alpha_{f1} \times \ell_2^2}{\alpha_{f2} \times \ell_1^2} \le 5 \qquad \text{where: } \alpha_f = \frac{\text{Ecb} \times \text{Ib}}{\text{Ecs} \times \text{Is}} = \frac{\text{Ib}}{\text{Is}}$$

$$\alpha_{f1} = \frac{4+4.5}{2} = 4.25$$

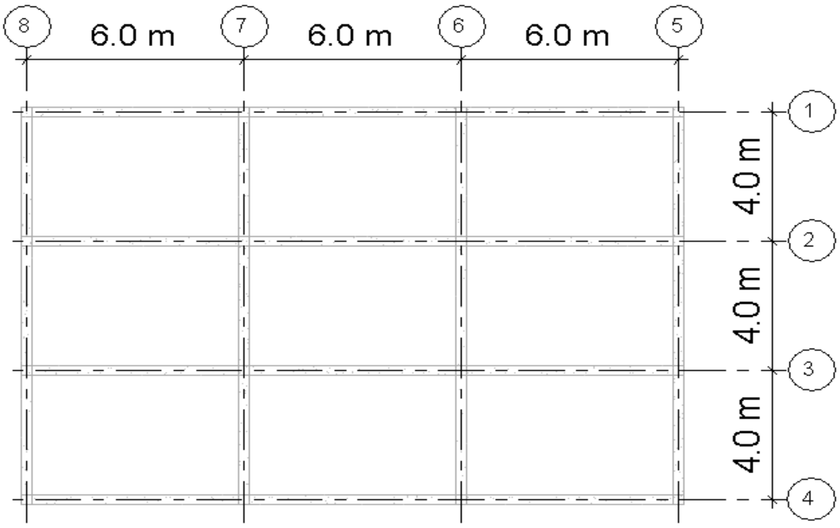
$$\alpha_{f2} = \frac{3.5+3}{2} = 3.25$$

$$0.2 \le \frac{4.25 \times 6^2}{3.25 \times 5^2} \le 5$$

$0.2 \le 1.88 \le 5$ O.K



Example: For the slab shown below check the limitation of the direct design method W_L=2 kN/m^2,W_D=3 kN/m^2 (including self-weight), α_{f1} for all beams= 3.5 α_{f2} for all beams=4



Solution:

1. **There must be a minimum of three continues spans in each direction**

   There are three spans in each direction **O.K**

2. **The panel must be rectangular with the ratio of the longer to the shorter spans within a panel not greater than 2.**

   $\frac{L}{S}=\frac{6}{4}$ =1.5 < 2 **O.K**

3. **Column offset shall not exceed 10 percent of the span in direction of offset from either axis between center-lines of successive columns.**

   All columns in each direction are on the same center line, and then the offset is equal to zero in each direction **O.K**

4. **Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span.**

   The successive span length is equal in each direction **O.K**

5. **All loads shall be due to gravity only (no lateral loads) and uniformly distributed over an entire panel.**

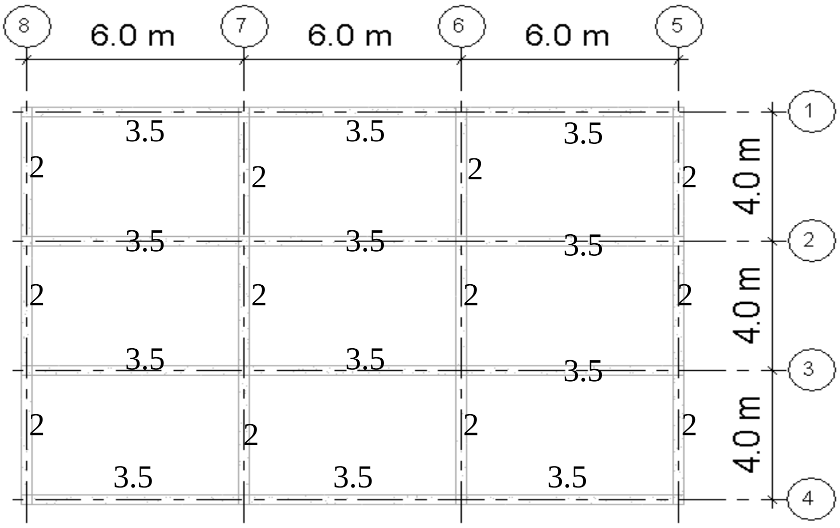
   There is nothing mentioned about lateral loads and all loads are due to gravity only **O.K**

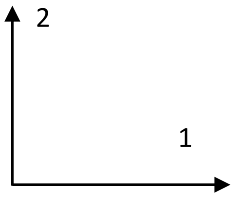
6. **Unfactored live load shall not exceed two times the unfactored dead load L.L ≤ 2D.L**.

   2 < 2*3 =2<6 **O.K**

- The first six of these criteria area easily satisfied by inception.

7.If beams are used between supports on all sides, the relative stiffness ratio of the beams in the two perpendicular directions must be between 0.2 and 0.5.





$0.2 \leq \frac{\alpha_{f1} * \ell_2^2}{\alpha_{f2} * \ell_1^2} \leq 5$ where: $\alpha_f = \frac{\text{Ecb}*\text{Ib}}{\text{Ecs}*\text{Is}} = \frac{\text{Ib}}{\text{Is}}$

All Panels have the same α_{f1} and α_{f2}

$$\alpha_{f1} = \frac{3.5+3.5}{2} = 3.5$$

$$\alpha_{f2} = \frac{2+2}{2} = 2$$

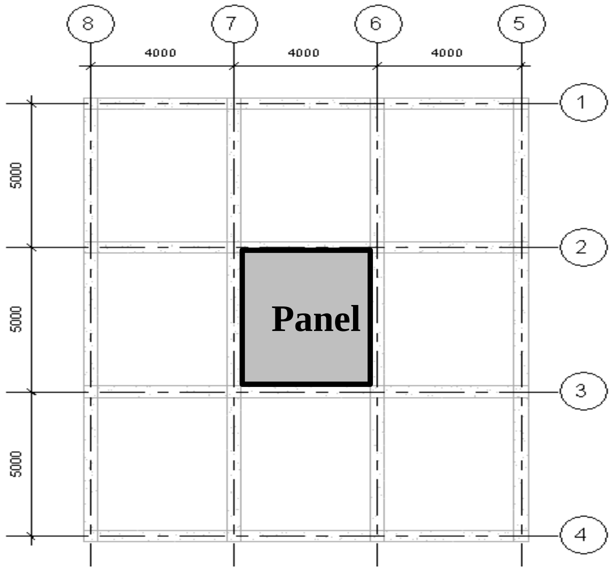
$$0.2 \leq \frac{3.5*4^2}{2*6^2} \leq 5$$

$0.2 \leq 0.78 \leq 5$ O.K

All the limitations of DDM are satisfied. ■

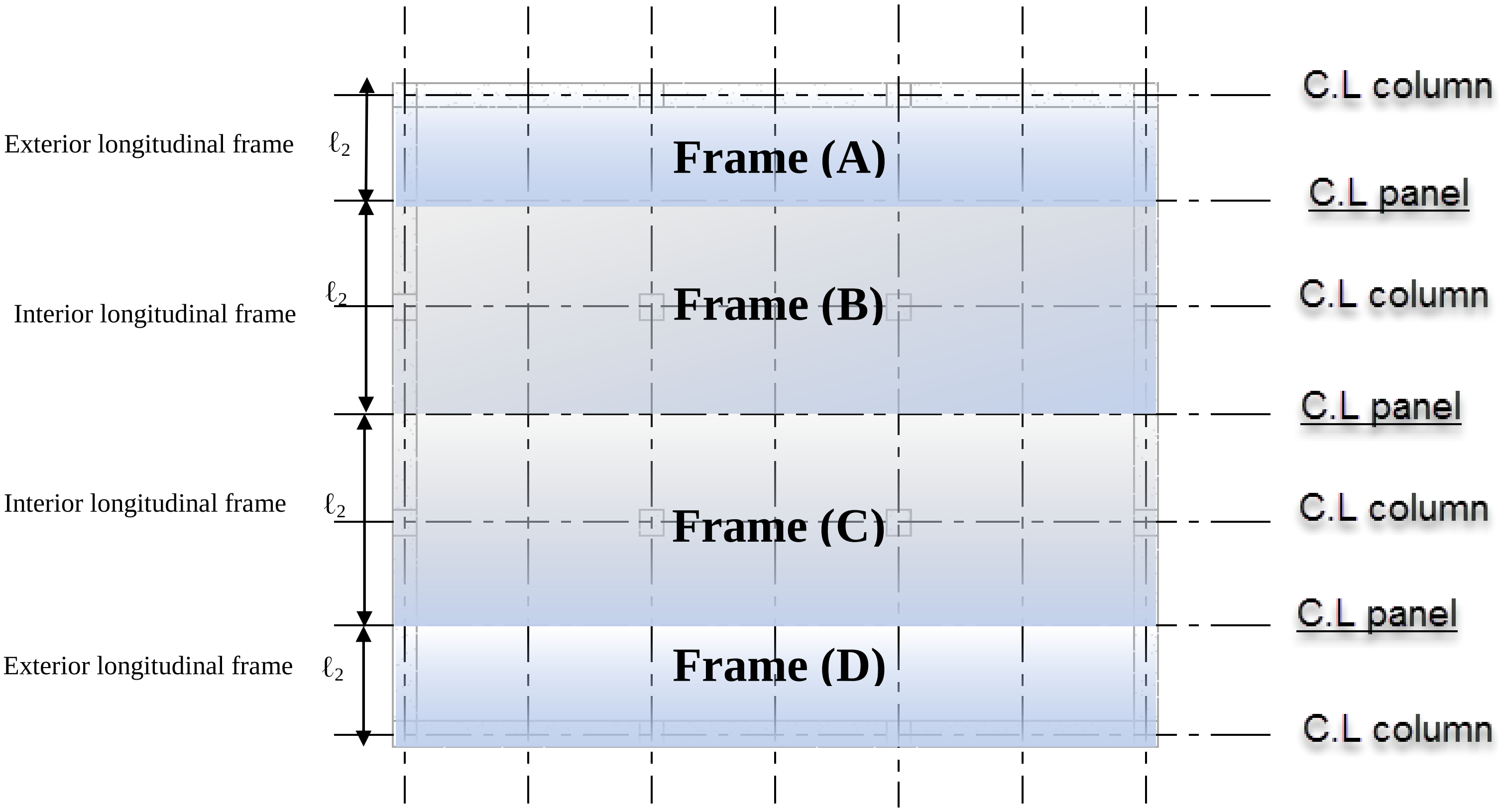
General Concept for design of Two-way Slab by DDM

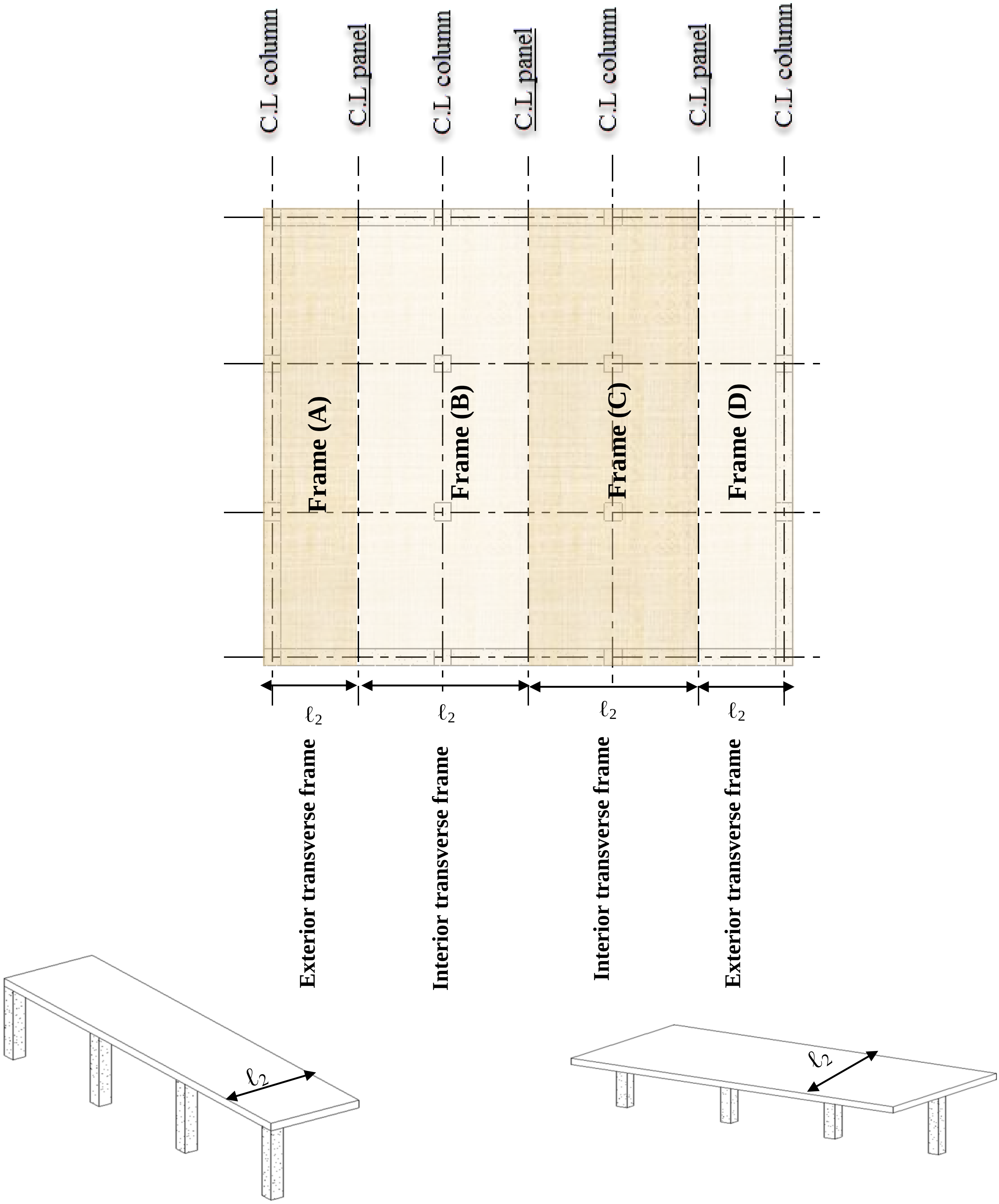
1. Divide the slab into panels by drawing center lines along the columns.



- **A panel** is bounded by column, beam, or wall **centerlines** on all sides (**ACI Code 8.4.1.7).**

2. Transform the three dimensional building into a simpler two dimensional frames by cutting the structure by imaginary vertical planes along the center lines of the panels (in each direction). Determine the design frames in the direction of the design **(usually mentioned in the question).**





Exterior frame

Interior frame

3. For each frame calculate the total static moment $\mathbf{M_o}$ For the exterior and interior frames.

   $M_o = \frac{w_u \ell_2 \ell_n^2}{8}$ **ACI Code (8.10.3.2)**

   Where:

   $W_u=1.2W_D+1.6W_L$

   ℓ_2 :Frame width.

   ℓ_n : clear span of ℓ_1 as follows: face to face of columns, capitals, brackets or wall but **not less than** 0.65 ℓ_1 .**ACI Code (8.10.3.2.1).**

   The clear span ℓ_n in the direction of moments is used.

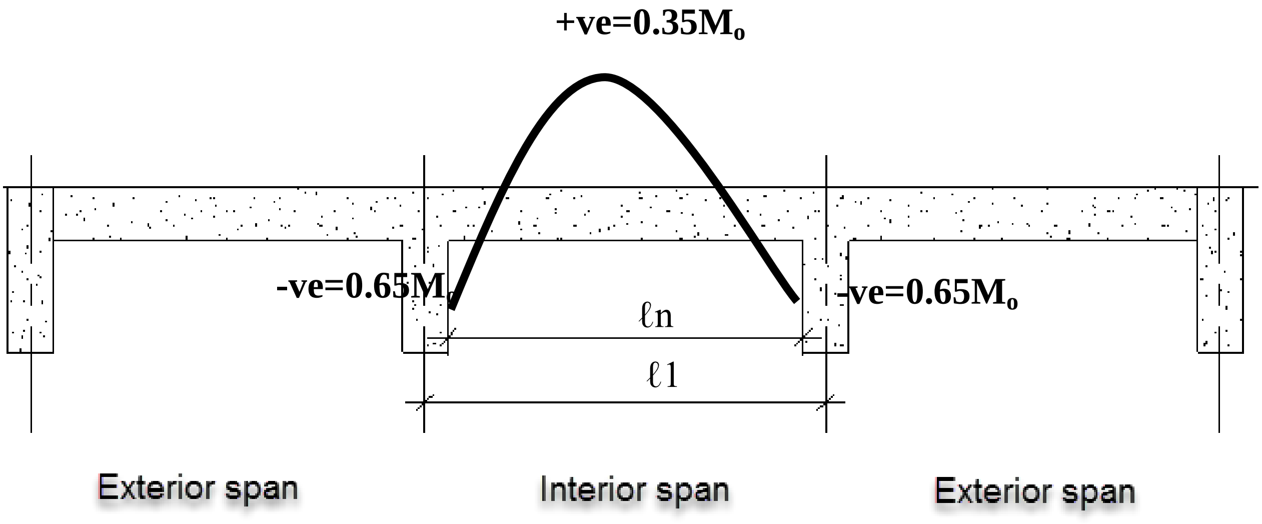
4. Longitudinal Distribute the total static moment $\mathbf{M_o}$ into **–ve** and **+ve** moments for each of the **end spans** and **interior spans. ACI Code (8.10.4.1).**
5. Lateral distribution of negative and positive moment to **column** and **middle strips** and to **beams**, if any.
   - The moment in column strip is distributed into the moment in the slab and the moment in beam (if there is any beam) this will be discussed later.

Longitudinal Distribute of Total Factored Static Moment Mo

A. Interior span

+ve moment at center of span=**$0.35M_o$**

-ve moment at face of support=**$0.65M_o$**



B. End (exterior) span

In the case of **end spans**, the apportionment of the total static moment among the **three** critical moment sections **(interior negative, positive , and exterior negative)** depend upon the flexural restraint provided for the slab by the exterior column or the wall, and depends also upon the presence or absence of beams on the column lines.

ACI Code Table 8.10.4.2. Specifies five alternative sets of moment distributions for **end span** shown in table below.

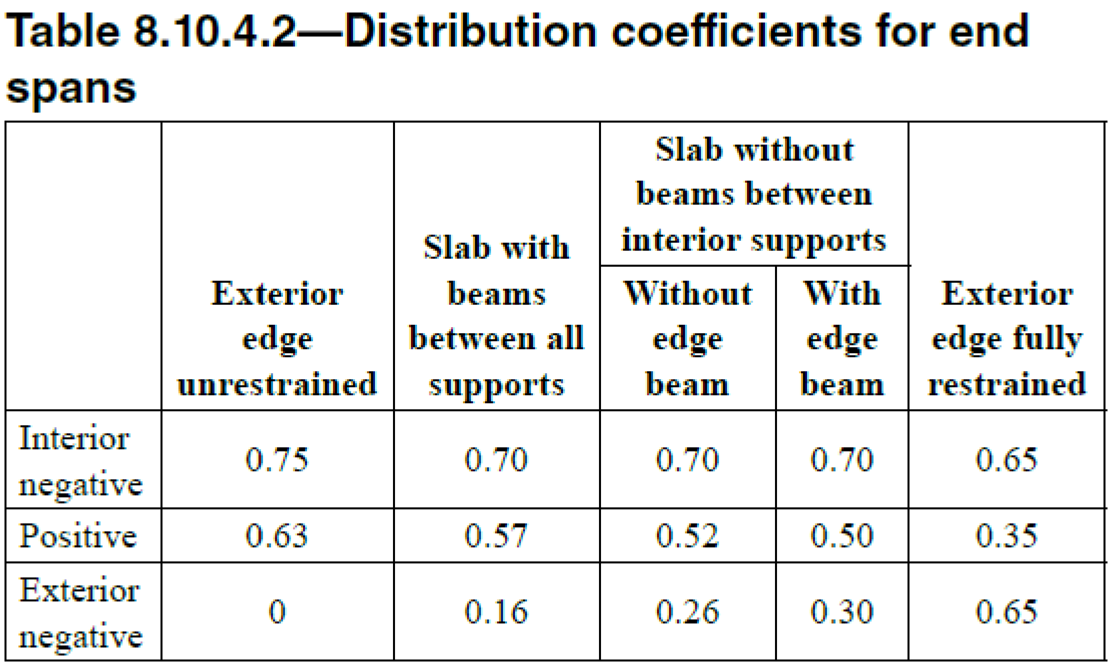
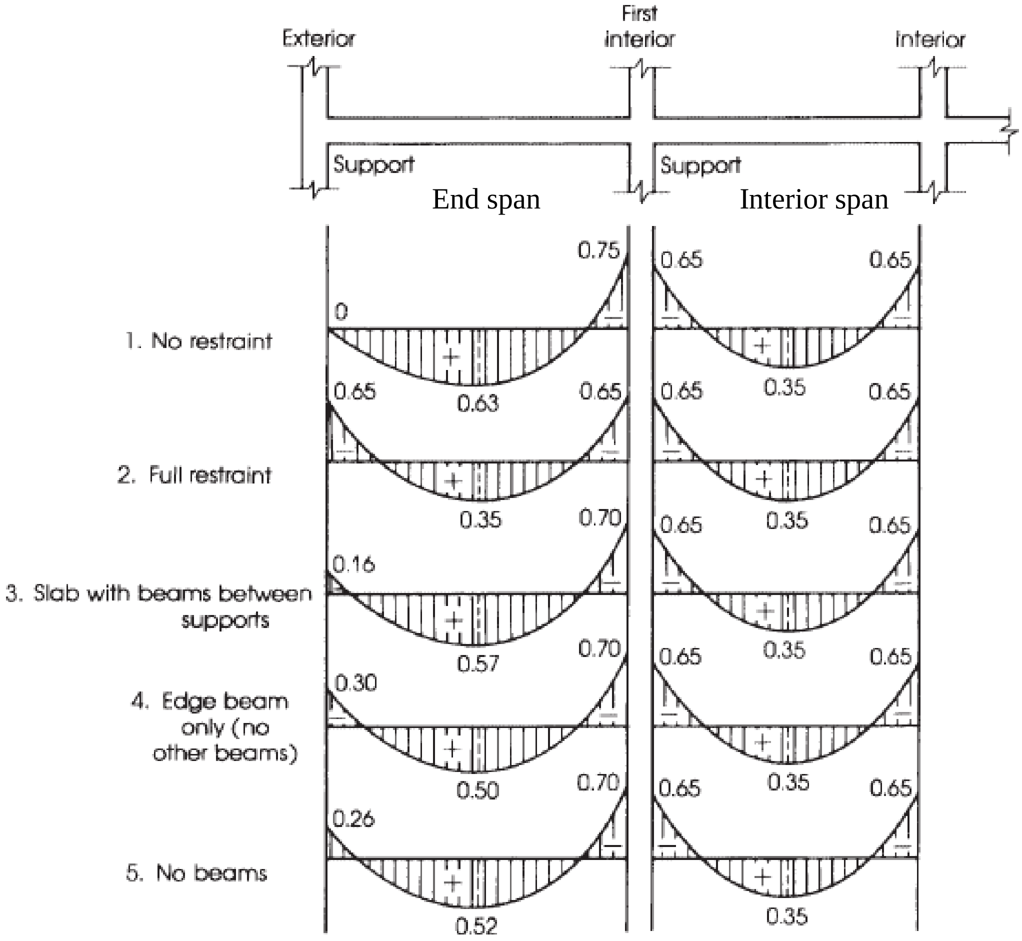


Table 8.10.4.2—Distribution coefficients for end spans

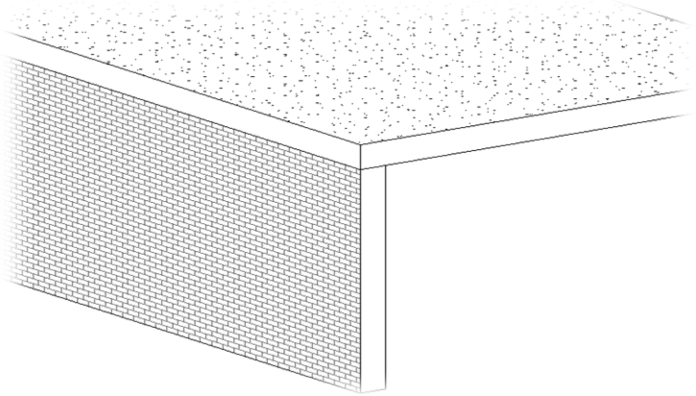
	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

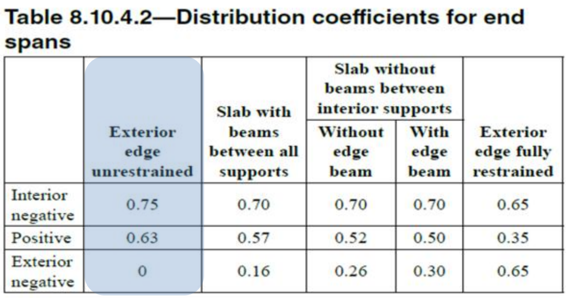


Longitudinal Distribution of total static moment into negative and positive span moments

- **ACI Code (8.10.4.4)** states that critical section for negative moment shall be **at the face** of **rectangular** supports.

- In **case 1 (exterior edge unrestrained)**, the exterior edge has no moment restraint, such as would be the condition with a masonry wall.





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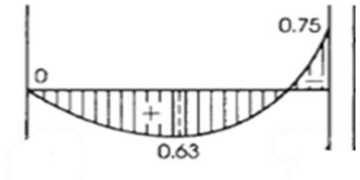
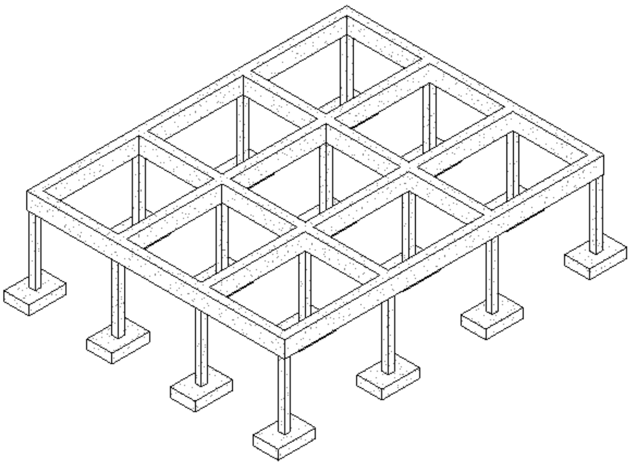
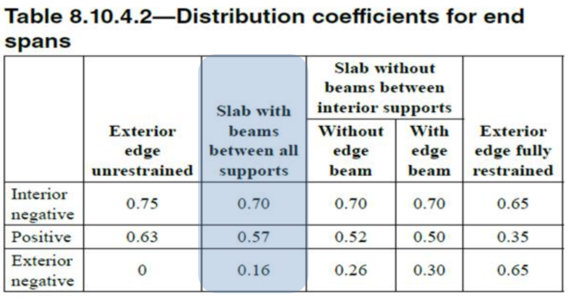


- In **Case 2 (slab with beams between all supports)** represents a two way slab with beams on all sides of the panels





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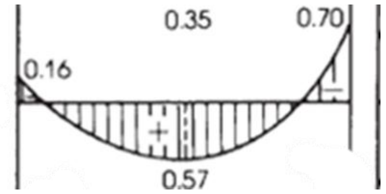
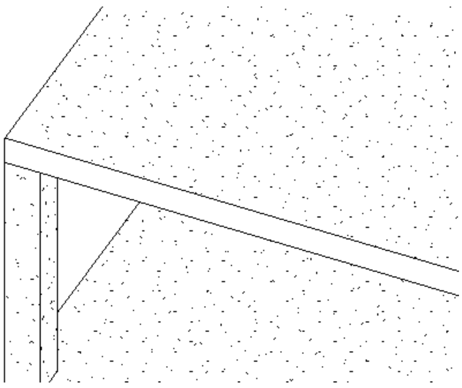
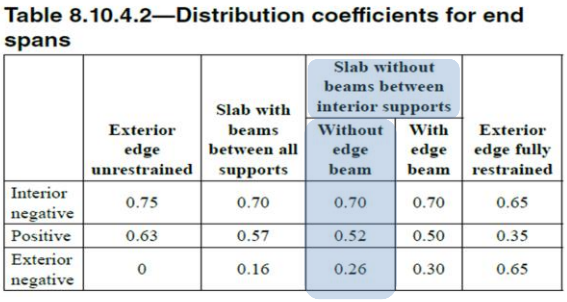


- In **Case 3 (slab without beams between interior supports and without edge beam)** it is like a flat plate, with no beams at all.





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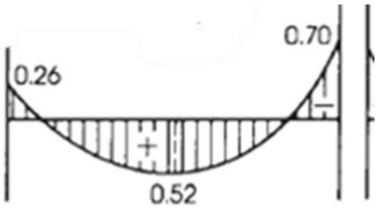
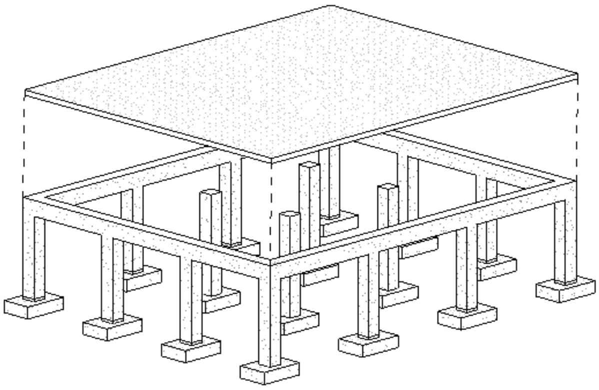
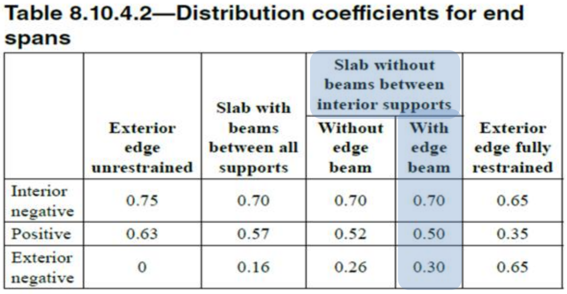


- In **Case 4 (slab without beams between interior supports and with edge beam)** it is like a flat slab with edge beams along exterior edge only.





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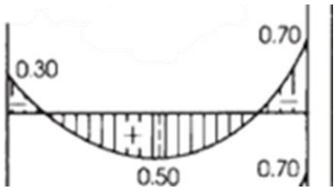
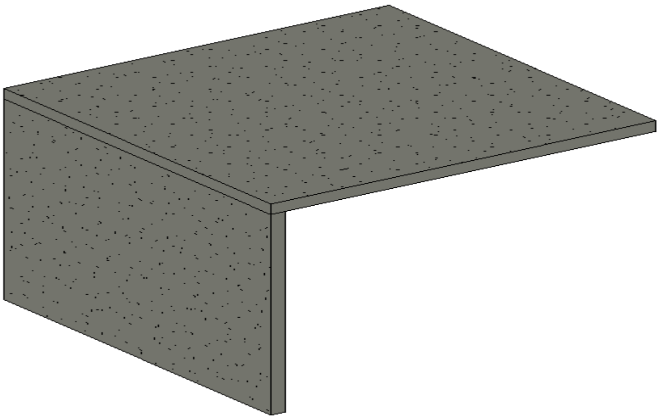
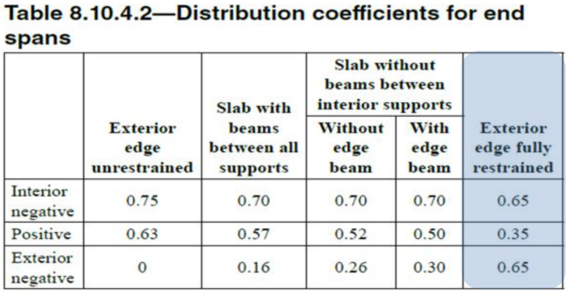


- Finally (**(Exterior edge fully restrained)** represents a fully restrained edge, such as that obtained if the slab is monolithic with very stiff reinforced concrete wall





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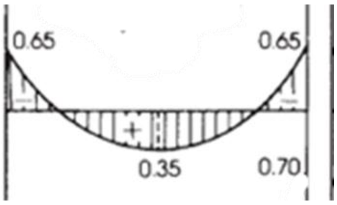
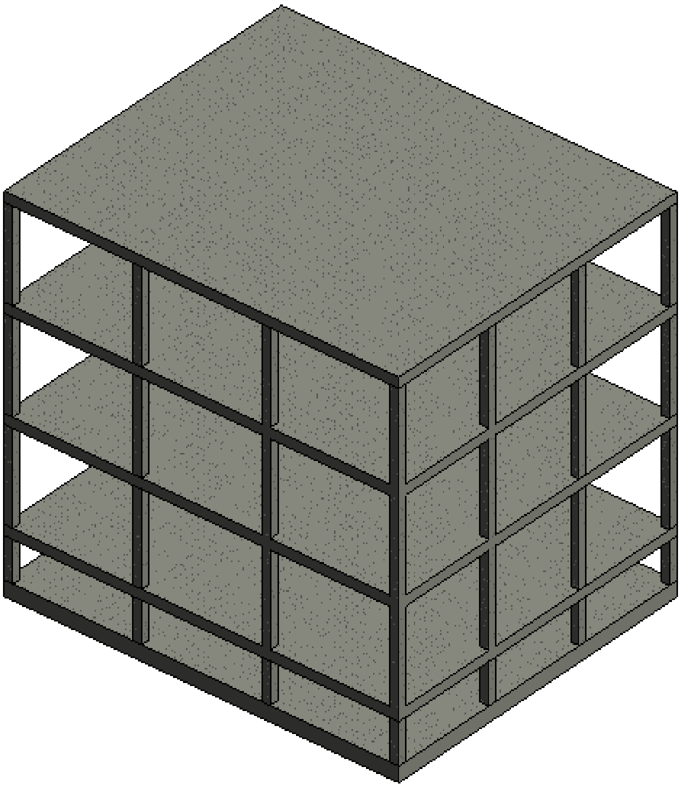
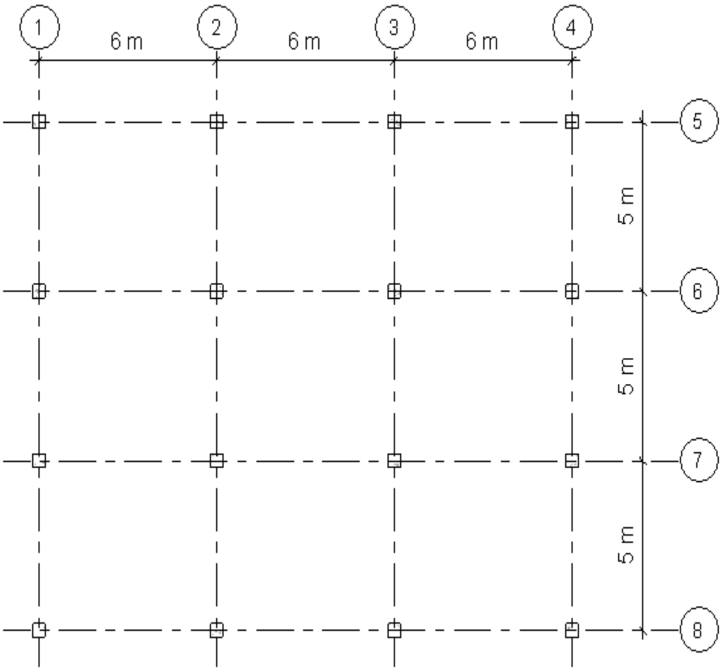


Example 1: For the flat plate slab shown below, the slab is subjected to W_L= 3 kN/m^2 and W_D=2.5 kN/m^2 (including self-weight), Columns dimensions are (400x400) mm. find:

1. Longitudinal distribution of the total static moment at factored loads for interior longitudinal frame.
2. Longitudinal distribution of the total static moment at factored loads for exterior transverse frame.





Solution:

$Wu=1.2W_D+1.6W_L$

$Wu=1.2(2.5)+1.6(3)=7.8$ kN/m^2

1. Longitudinal distribution of the total static moment at factored loads for interior longitudinal frame.

$M_o=\frac{Wu*\ell n^2*\ell 2}{8}$

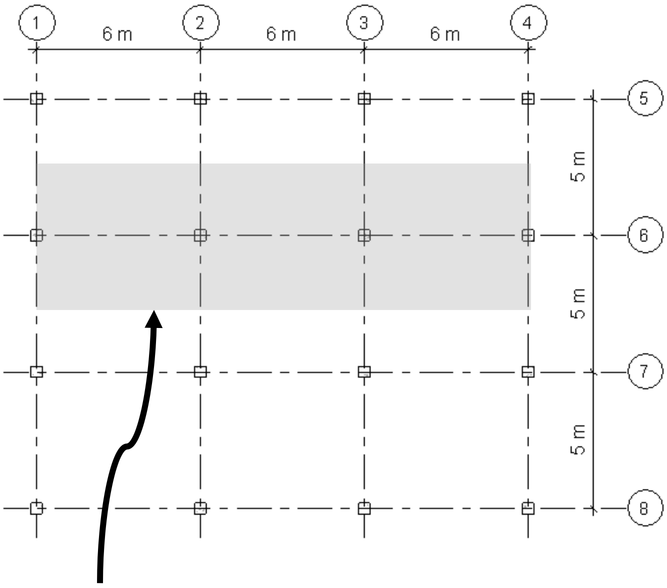
ℓ_n=6-0.4=5.6 m or 0.65 *ℓ_1=0.65*6=3.9 m

Use ℓ_n=5.6 m

$\ell_2=(\frac{5}{2}+\frac{5}{2})$=5 m

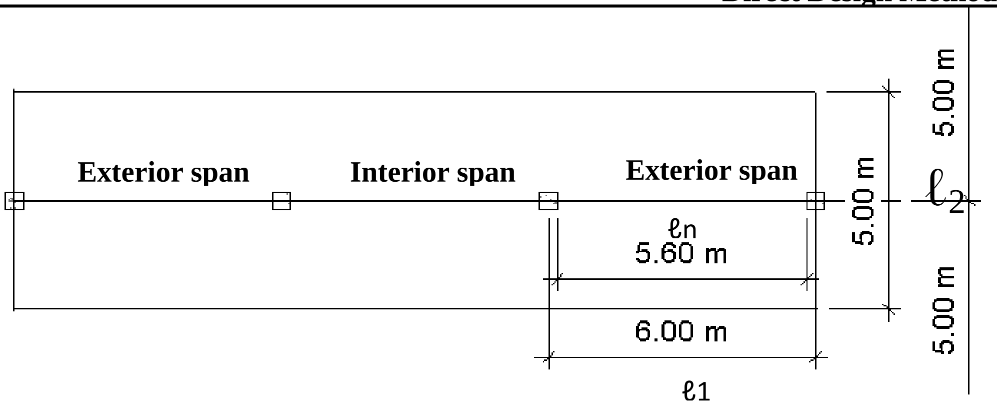
$M_o=\frac{7.8*5.6^2*5}{8}$

= 152.88 kN.m



Longitudinal interior frame

-Distribute the static moment **(152.88 kN.m)** into **+ve** and **–ve** moments.



Longitudinal distribution

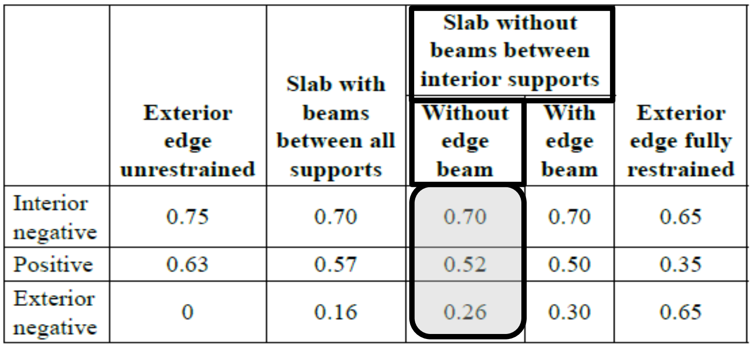
For interior span:

+ve moment at center of span=**0.35M$_o$** =0.35*152.88=53.508 kN.m

-ve moment at face of support=**0.65M$_o$ =**0.65*152.508=99.37 kN.m

For exterior (end) span:

Table 8.10.4.2—Distribution coefficients for end spans



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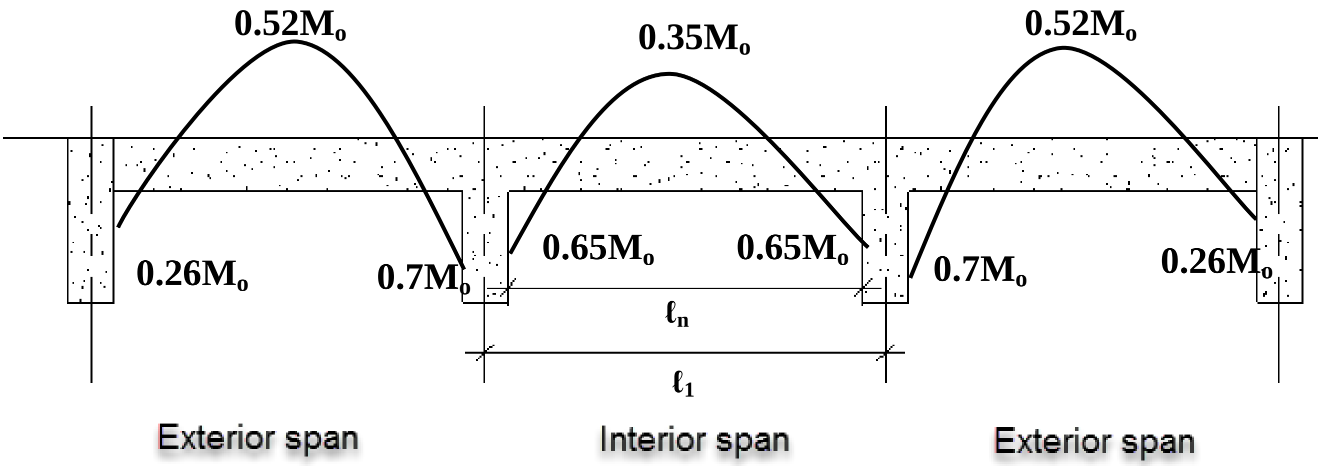
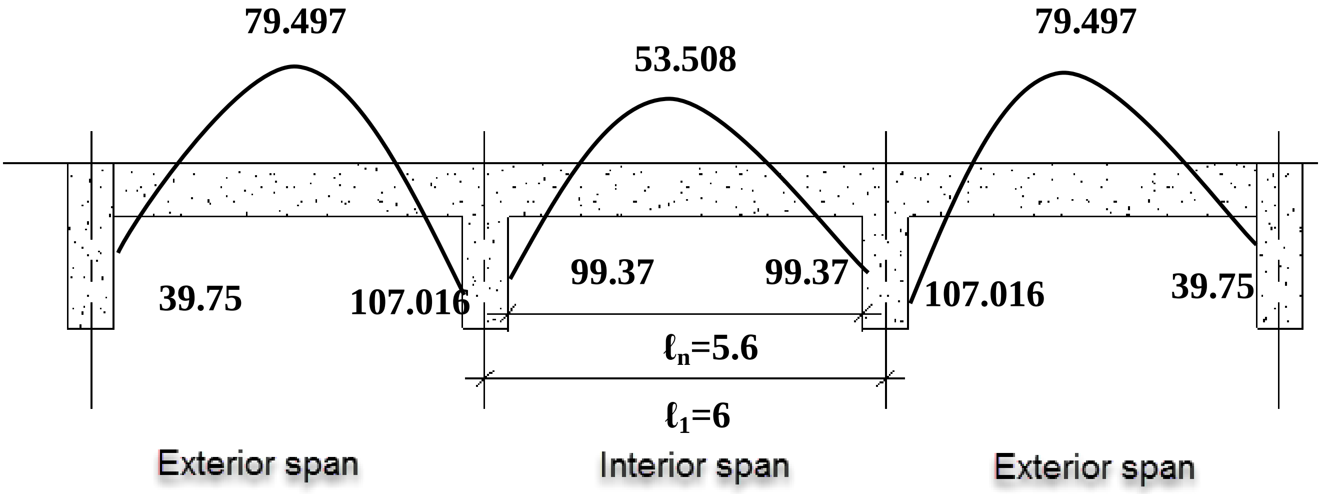
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

Moment for **Interior** negative at the face of support=**0.7*M$_o$**=0.7*152.88=107.016 kN.m

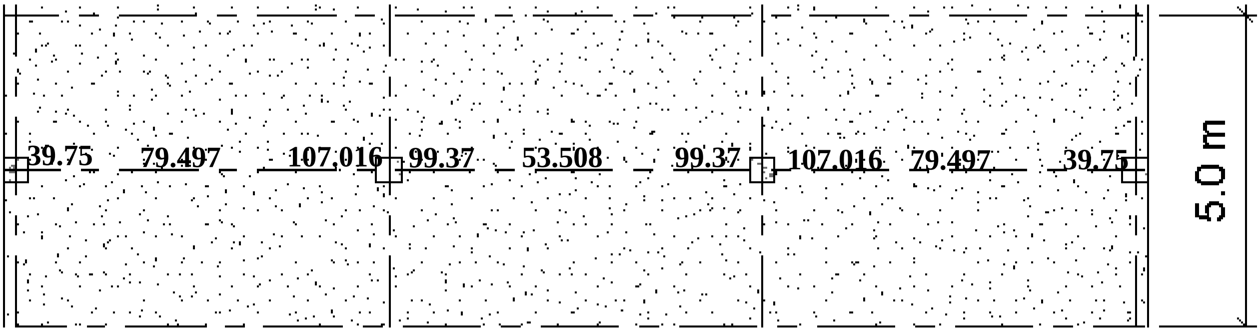
Positive moment at the **center** of span=**0.52*M$_o$**=0.52*152.88=79.497 kN.m

Moment for **exterior** negative at face of support=**0.26*M$_o$**=0.26*152.88=39.7488 kN.m





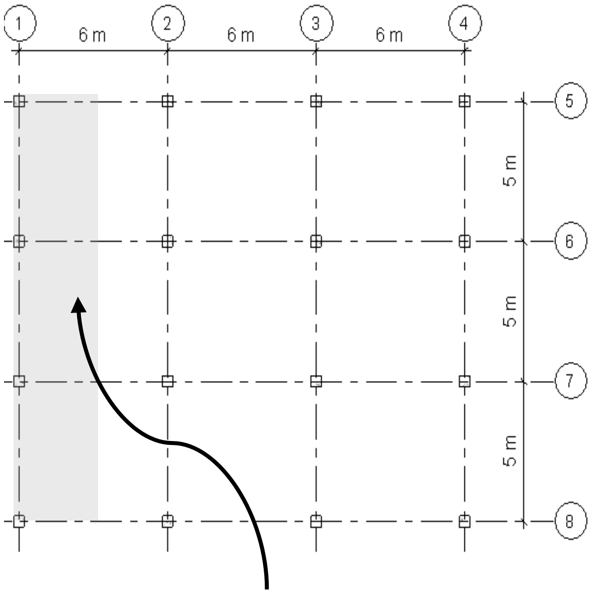
Longitudinal distribution of the total static moment (kN.m) at factored loads for interior longitudinal frames.



Exterior span **Interior span** **Exterior span**

Longitudinal distribution of the total static moment (kN.m) at factored loads for interior longitudinal frames.

2. **Longitudinal distribution of the total static moment at factored loads for exterior transverse frame.**



Transverse exterior frame

$Wu=7.8\ kN/m^2$

$M_o= \frac{Wu*\ell n^2*\ell 2}{8}$

$\ell_n=5-0.4=4.6$ m or $0.65*\ \ell_1=0.65*5=3.25$ m

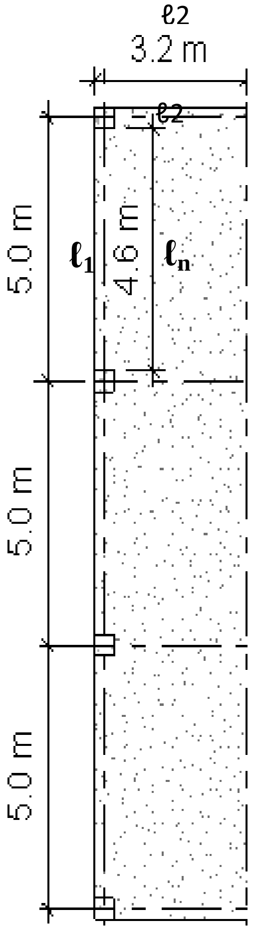
Use $\ell_n=4.6$ m

$\ell_2= (\frac{6}{2}+\frac{0.4}{2})=3.2$ m

$M_o = \frac{7.8*4.6^2*3.2}{8}$

= 66.019 kN.m

-Distribute the span moment **(66.019 kN.m)** into **+ve** and **–ve** moments.



Longitudinal distribution

For interior span:

+ve moment at center of span=$\mathbf{0.35M_o}$ =0.35*66.019=23.1 kN.m

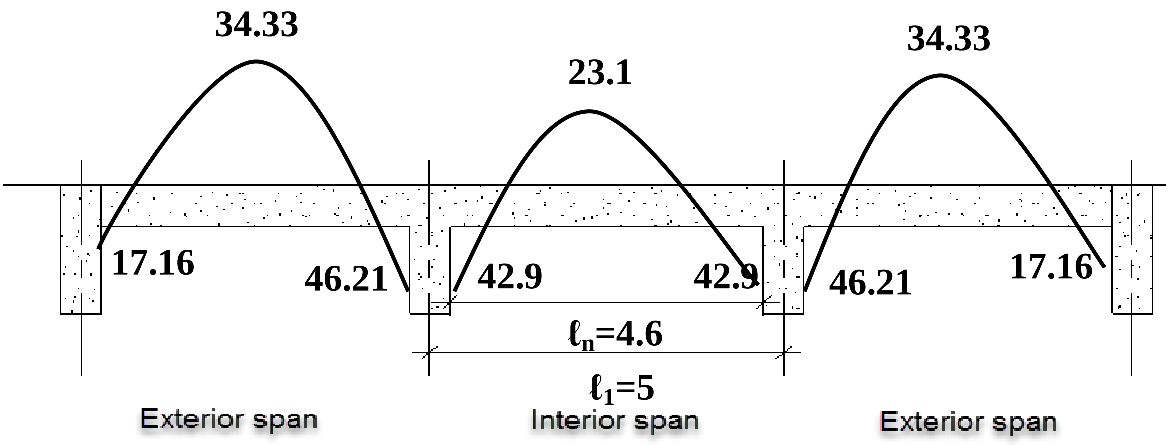
-ve moment at face of support=$\mathbf{0.65M_o}$ =0.65*66.019=42.9 kN.m

For exterior (end) span:

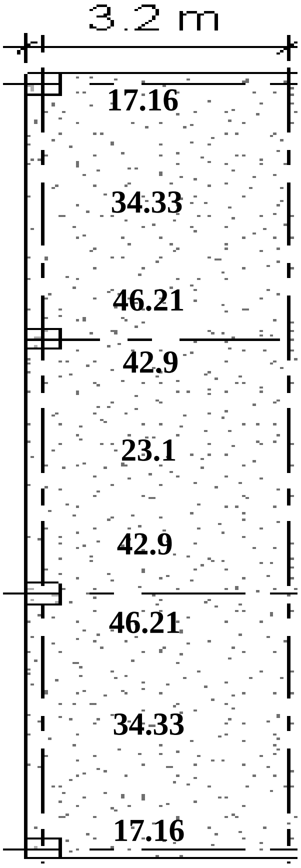
Interior negative moment =$\mathbf{0.7*M_o}$=0.7*66.019=46.21 kN.m

Positive moment =$\mathbf{0.52*M_o}$=0.52*66.019=34.33 kN.m

Exterior negative moment=$\mathbf{0.26*M_o}$=0.26*66.019=17.16 kN.m



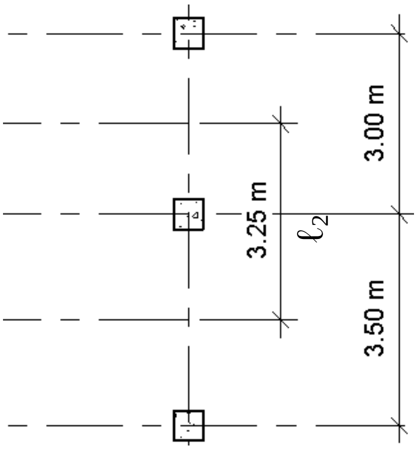
Longitudinal distribution of static moment



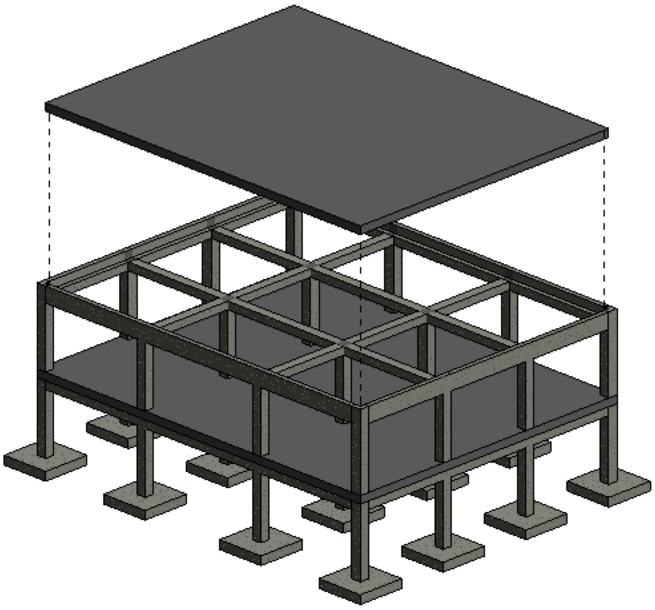
- Section (**8.10.3.2.3) of ACI Code** states that if the transverse span of panels on either side of the centerline of supports varies, ℓ_2 shall be taken as the **average of adjacent transverse spans.**

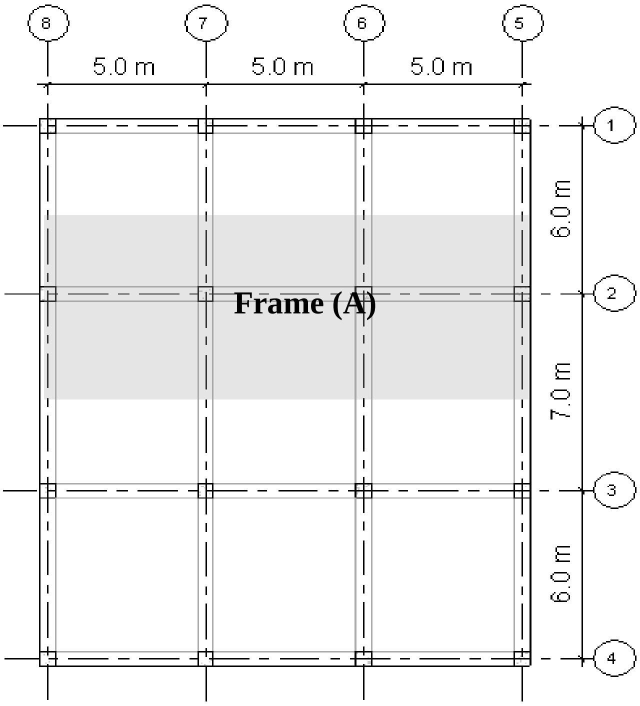
Example: Find ℓ_2 for figure shown

$$\ell_2 = \left(\frac{3}{2} + \frac{3.5}{2}\right) = 3.25\ m$$



Example 2: For the multi-story building shown in figures below, for frame (A) find the longitudinal distribution of the static moment at factored loads. Slab thickness is (200) mm Wu=12.96 kN/m^2 and column dimensions are (500 x 500) mm.





Solution:

W_u=12.96 kN/m^2

For Frame (A)

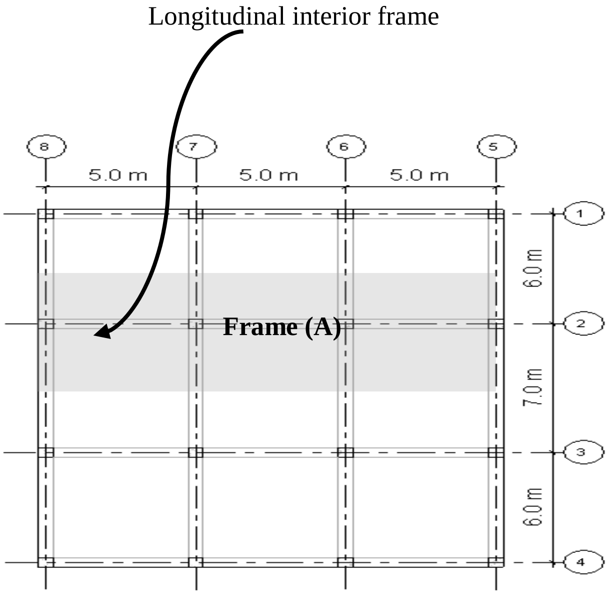
$$M_o= \frac{Wu*\ell n^2*\ell 2}{8}$$

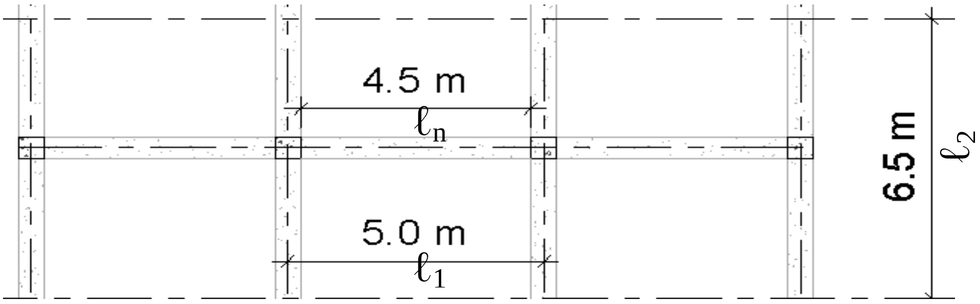
ℓ_n=5-0.5=4.5 m or 0.65 *ℓ_1=0.65*5=3.25 m

Use ℓ_n=4.5 m

ℓ_2= $(\frac{6}{2}+\frac{7}{2})$=6.5 m **ACI Code (8.10.3.2.3)**

$$M_o = \frac{12.96*4.5^2*6.5}{8} = 213.23 \text{ kN.m}$$





Longitudinal Distribution

For interior span:

+ve moment at center of span=**0.35M$_o$** =0.35*213.23=74.63 kN.m

-ve moment at face of support=**0.65M$_o$ =**0.65*213.23=138.6 kN.m

For exterior (end) span:

Interior negative moment =**0.7*M$_o$**=0.7*213.23=149.26 kN.m

Positive moment =**0.57*M$_o$**=0.57*213.23=121.54 kN.m

Exterior negative moment=**0.16*M$_o$**=0.16*213.23=34.12 kN.m

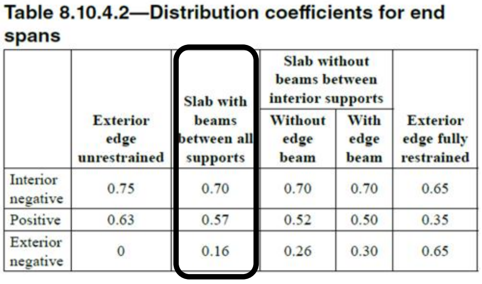
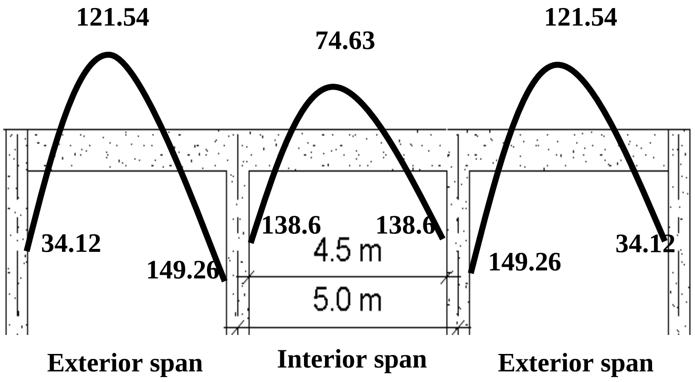


Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



Example 3: for the longitudinal interior frame (A) of the flat plate floor shown in figure below by using direct design method find:

Longitudinal distribution of the static moment at factored loads

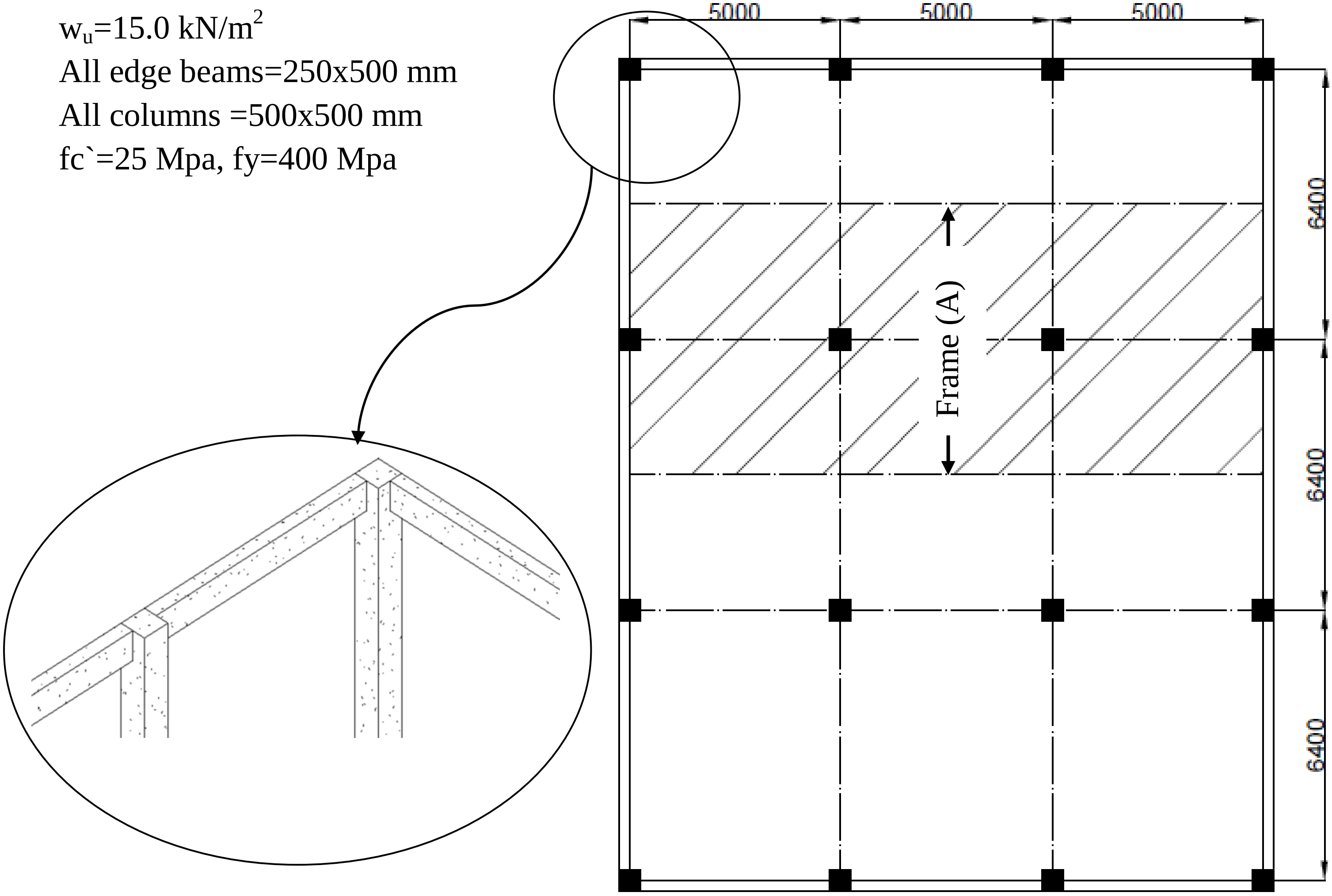
Slab thickness = 200 mm, d=165 mm

w_u=15.0 kN/m^2

All edge beams=250x500 mm

All columns =500x500 mm

fc`=25 Mpa, fy=400 Mpa



Solution:

For frame (A)

ℓ_1=5000 mm

ℓ_n=5000-500=4500 mm or 0.65 ℓ_1

ℓ_n=**4500**mm or 3250mm use larger

ℓ_2= $(\frac{6400}{2}+\frac{6400}{2})$ = 6400 mm

M_o**=** $\frac{Wu*\ell n^2*\ell 2}{8}$

M_o= $\frac{15*4.5^2*6.4}{8}$ =243 kN.m

Longitudinal distribution of total static moment at factored loads

Interior span:

Neg. Mu=**0.65M_o**=0.65*243=157.95 kN.m

Pos.Mu=**0.35M_o**=0.35*243=85.05kN.m

End Span:

Interior Neg.Mu=**0.7M_o**=0.7*243=170.1 kN.m

Pos.Mu=**0.5M_o**=0.5*243=121.5 kN.m

Exterior Neg.Mu=**0.3M_o**=0.3*243=72.9 kN.m

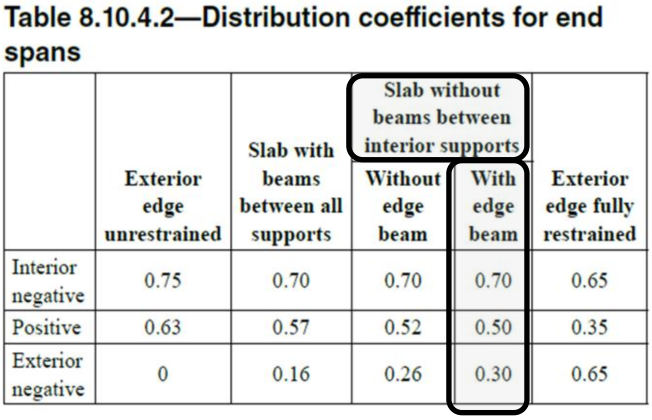
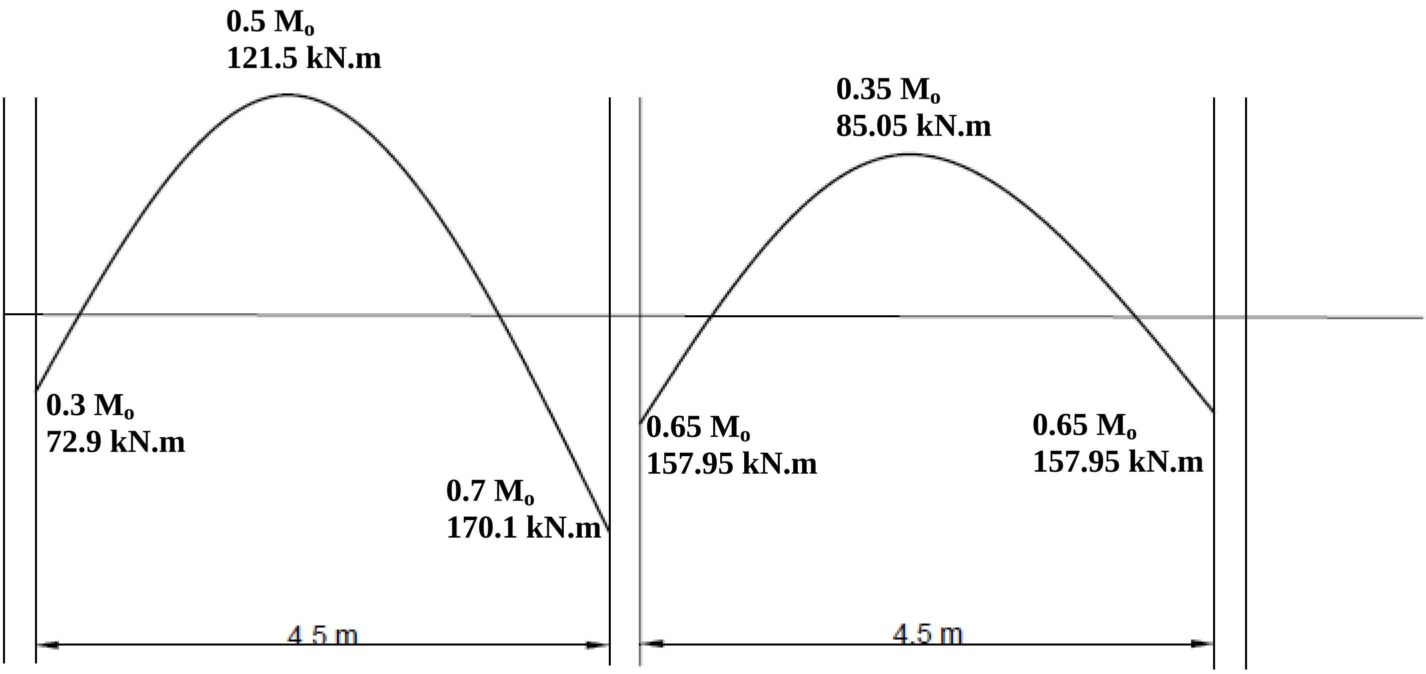


Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



Home Work: For the transverse interior frame (Frame C) of the flat plate floor with edge beams shown in Figure below, by using the direct design method find:

Longitudinal distribution of total static moment at factored loads

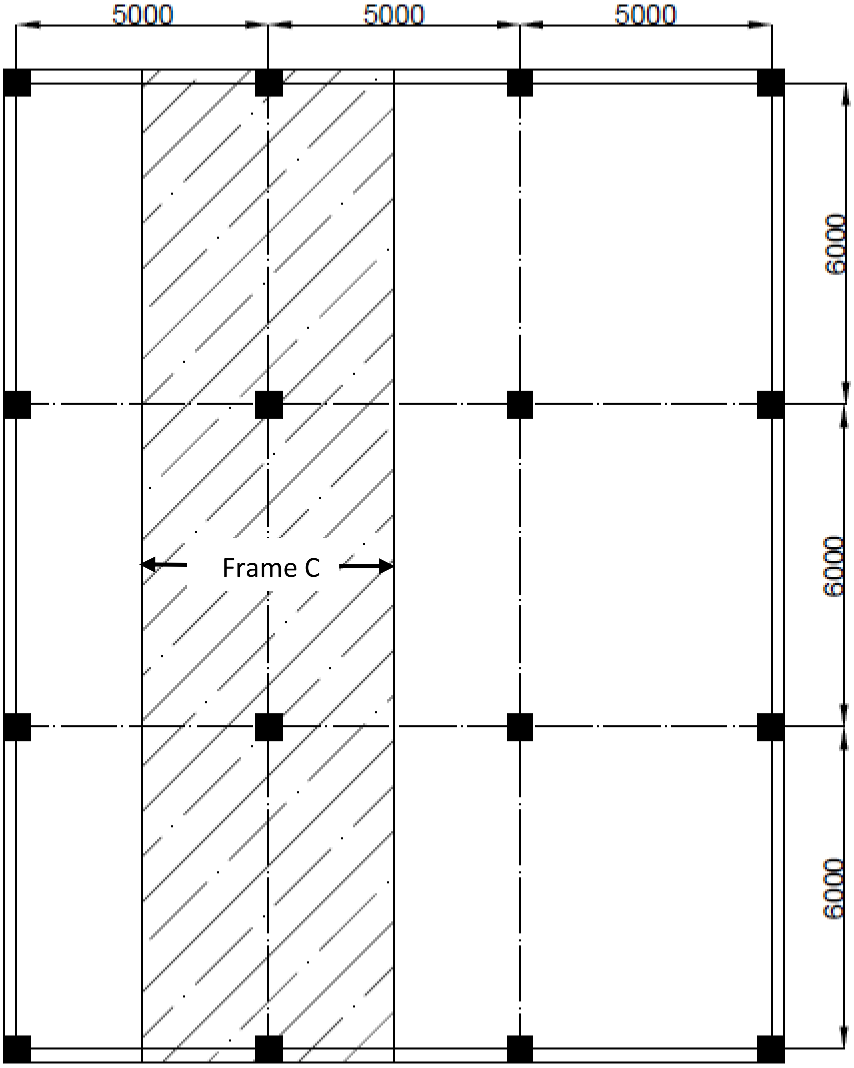
Slab thickness = 180 mm, d=150 mm

w_u=14.0 kN/m^2

All edge beams=250x500 mm

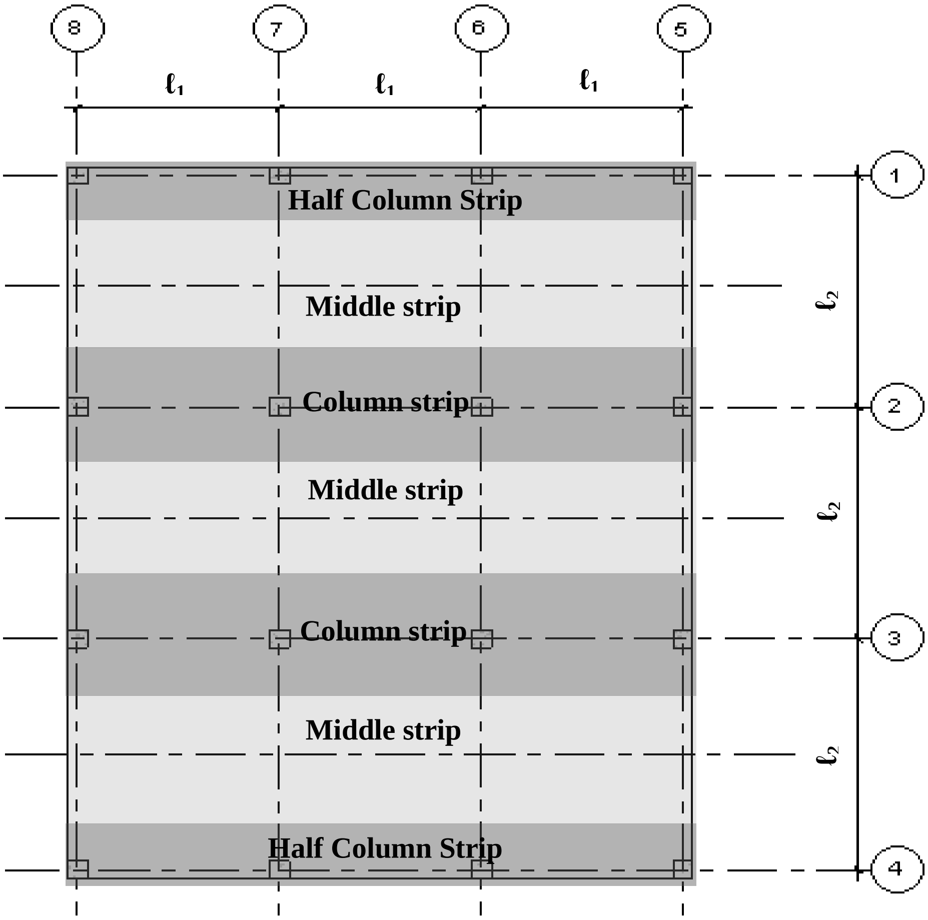
All columns =500x500 mm

fc`=24 Mpa, fy=400 Mpa

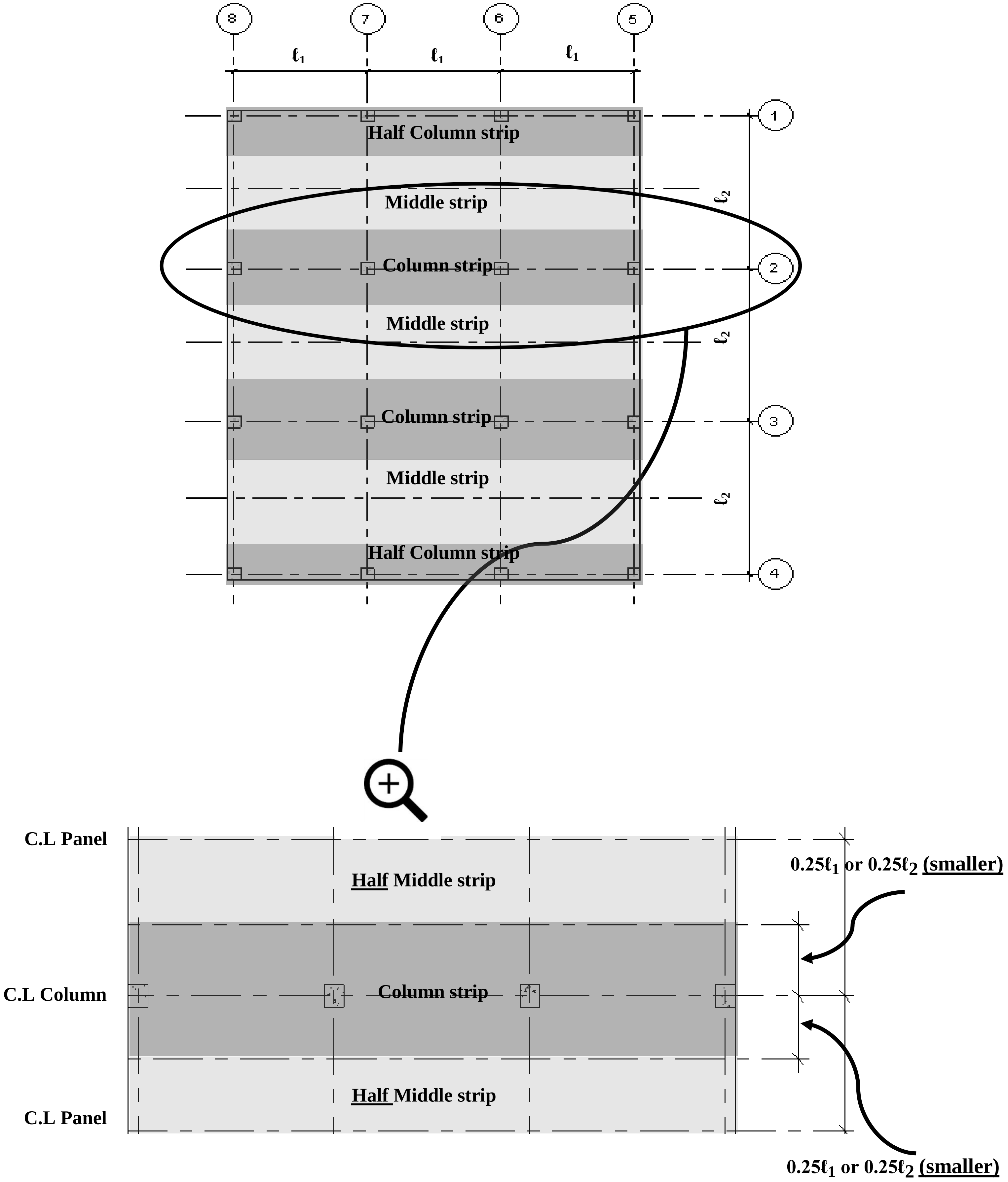


Lateral distribution of longitudinal moment

- Having distributed the total factored static moment **(M_o)** to the **positive** and **negative** moment as described previously, the designer still must distribute each of positive and negative moments into the **column strip** and **middle strip** for each design frame.
- The longitudinal moment values mentioned in the previous section are for the **entire width** of the equivalent building frame. This frame width is the sum of the widths of **two half column strips** and **two half-middle strips** of two adjacent panels, as shown in Fig. below The lateral (transverse) distribution of the longitudinal moments to the middle and column strips is a function of the **1. Relative stiffness of parallel beam (α_f)**, **2. The ratios of ℓ_2/ℓ_1**, **3. Torsional stiffness of torsional member (C)** and **4. Relative stiffness for Torsional member (βt)**.



- A **column strip** is a design strip with a width on **each side** of a **column centerline** equal to **lesser** of **$0.25\ell_1$** or **$0.25\ell_2$**. A column strip should include beams with in if presented. **ACI Code (8.4.1.5).**
- **A middle strip** is a design strip bounded by **two column strip. ACI Code (8.1.4.6)**


8
7
6
5
ℓ1
ℓ1
ℓ1
1
2
3
4
ℓ2
ℓ2
ℓ2
Half Column strip
Middle strip
Column strip
Middle strip
Column strip
Middle strip
Half Column strip
C.L Panel
Half Middle strip
0.25ℓ1 or 0.25ℓ2 (smaller)
C.L Column
Column strip
C.L Panel
Half Middle strip
0.25ℓ1 or 0.25ℓ2 (smaller)

A. Interior negative factored moments in the column strip

Column strip shall be proportioned to resist the following portions in percent of interior negative factored moments **ACI Code (8.10.5.1).**

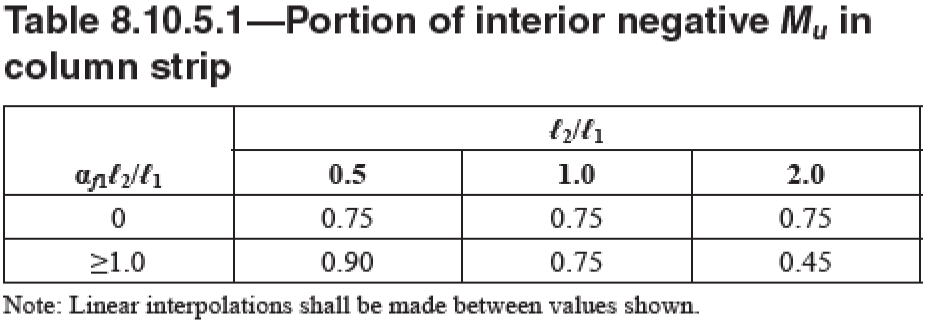


Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	**1.0**	**2.0**
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Note: The linear interpolation for transverse distribution of interior negative moment between the column strips can be based on the following equation:

-Interior C.S $_{\text{coefficient}}$**%**$=75 + 30(\alpha_{f1}\frac{\ell_2}{\ell_1}) \times (1-\frac{\ell_2}{\ell_1})$

- $\boldsymbol{\alpha_{f1}}$ **:** (value of $\boldsymbol{\alpha_{f1}} = \frac{\boldsymbol{Ib}}{\boldsymbol{Is}}$ in the **direction** of the **moment**)
- $\frac{\ell_2}{\ell_1}$: (center to center of support)
- When there is **no beam** in the direction of α_{f1}= 0
- When $(\alpha_{f1}\frac{\ell_2}{\ell_1})$ >1, use 1 in above equation.

Negative $M_{C.S}$ interior=C.S *-M interior

- The portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips **ACI Code (8.10.6.1).**

Negative $M_{M.S}$ interior= M interior- Negative $M_{C.S}$

B. Positive factored moments in the column strip (for all spans)

Column strips shall be proportioned to resist the following portions in percent of positive factored moments **ACI Code (8.10.5.5).**

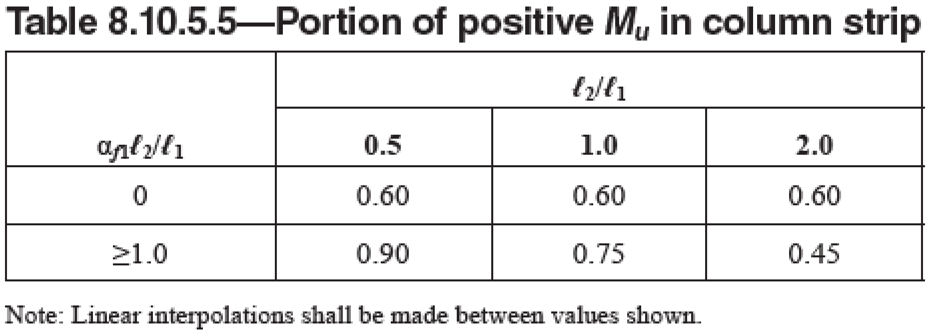


Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	**1.0**	**2.0**
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Note: The linear interpolation for transverse distribution of positive moment between the column strips can be based on the following equation:

$\underline{\mathbf{+C.S}}_{\text{coefficient}}\ \mathbf{\%} = 60 + 30(\alpha_{f1}\frac{\ell_2}{\ell_1}) \times (1.5-\frac{\ell_2}{\ell_1})$

- α_{f1} : (value of $\alpha_{f1} = \frac{Ib}{Is}$ in the **direction** of the **moment**)
- $\frac{\ell_2}{\ell_1}$: (center to center of support)
- When there is **no beam** in the direction of $\alpha_{f1} = 0$
- When $(\alpha_{f1}\frac{\ell_2}{\ell_1}) > 1$, use 1 in above equation.

$$\boxed{\textbf{Positive } M_{C.S} = C.S \times +M}$$

$$\boxed{\textbf{Positive } M_{M.S} = +M - \textbf{Positive } M_{C.S}}$$

C. Exterior negative factored moments in the column strip

Column strip shall be proportioned to resist the following portions in percent of exterior negative factored moment.

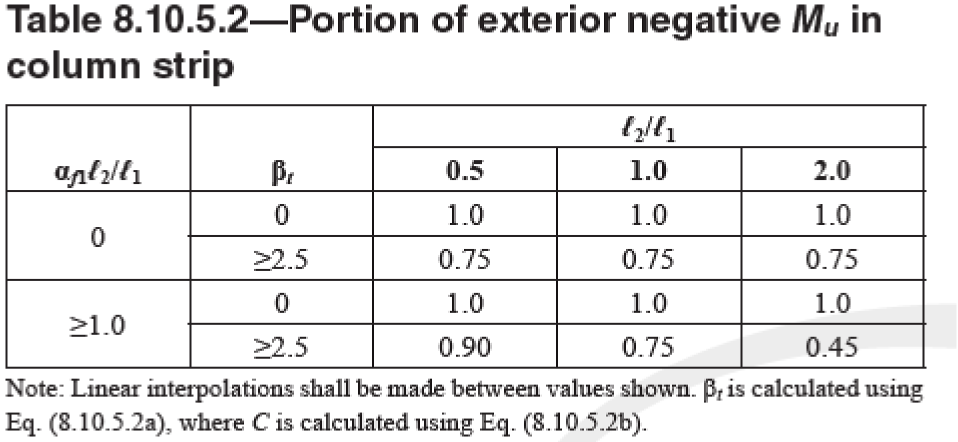


Table 8.10.5.2—Portion of exterior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	β_t	ℓ_2/ℓ_1		
		0.5	**1.0**	**2.0**
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

Note: the linear interpolation for transverse distribution of exterior negative moment between the column strip and middle strip can be based on the following equation:

-Exterior C.S $_{\text{coefficient}}$% =100 - 10βt + 12 βt $(\alpha_{f1}\frac{\ell_2}{\ell_1}) \times (1-\frac{\ell_2}{\ell_1})$

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad (8.10.5.2a)$$

$$C = \Sigma\left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3} \qquad (8.10.5.2b)$$

Negative $M_{C.S}$ Exterior=C.S*-M

Negative $M_{M.S}$ =M Exterior - Negative $M_{C.S}$

Notes: ACI Code **R (8.10.5.2)**

1.β_t = 0 (for flat plat without edge beam and brick wall)

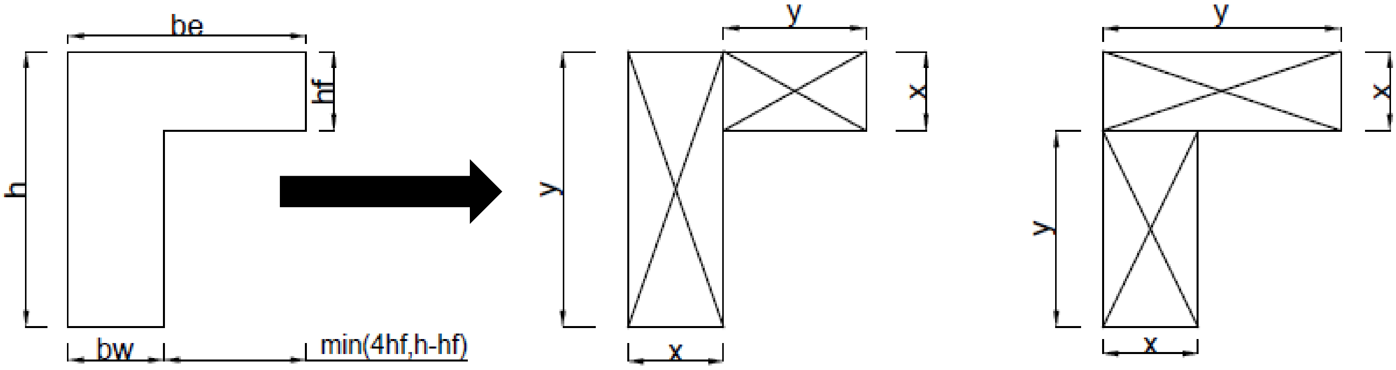
β_t = 2.5 (reinforced concrete wall)

2. Where walls are used as supports along column lines, they can be regarded as very stiff beams with an $\alpha_{f1}\frac{\ell_2}{\ell_1}$ >1

Torsional constant (C)

The constant **C** is calculated by **dividing** the section into its component **rectangles**, each having **smaller dimension x** and **larger dimension y**, and summing the contributions of all the parts by mean of the equation:

$$C=\sum(1-0.63\frac{x}{y})(\frac{x^3y}{3})$$

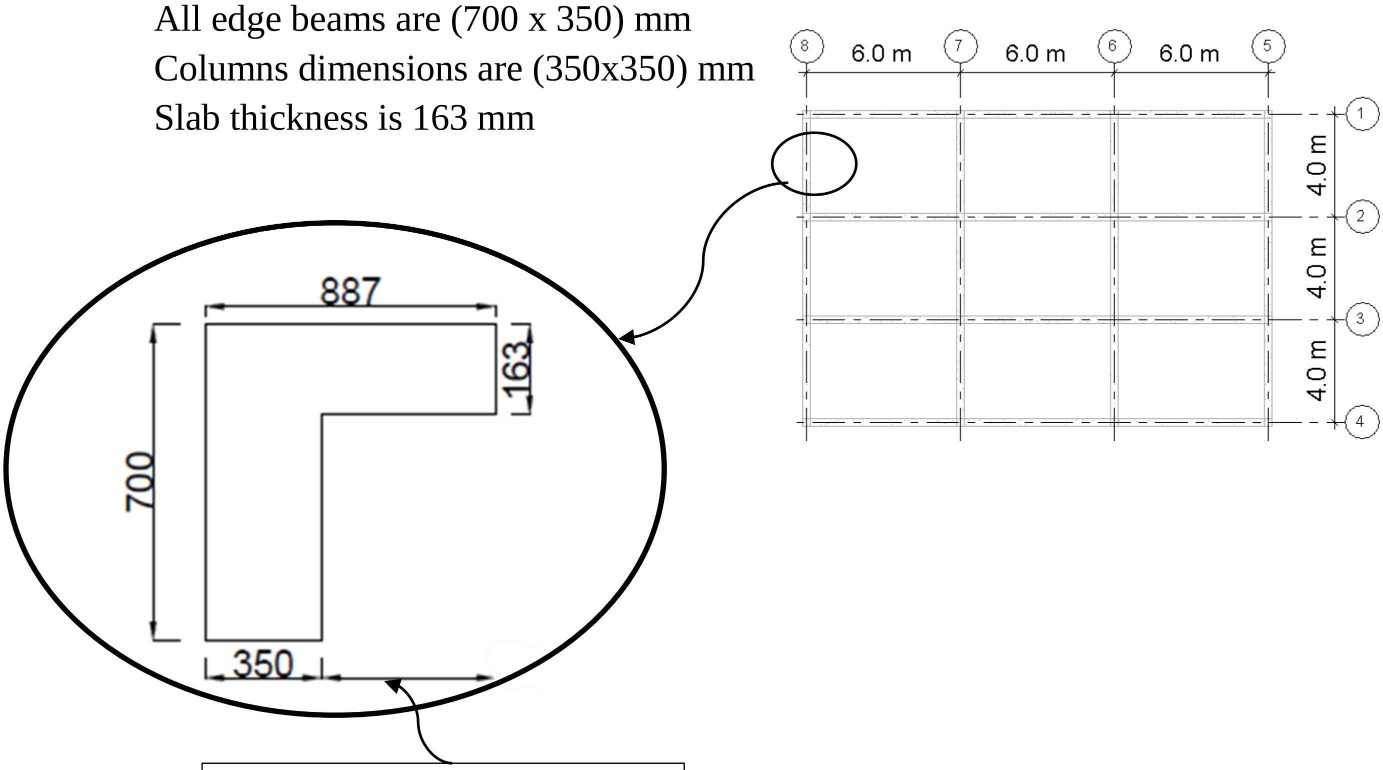


(a) (b) (c)

- The maximum value normally is obtained when wider rectangle is made as long as possible. Thus, the rectangular chosen in **Figure (b)** above will give **larger** C than those in **Figure (c)**.

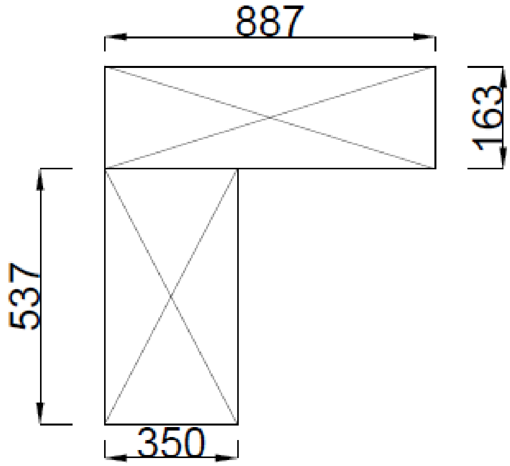
Example: compute the **torsional constant C** for the edge beam in figure below:

All edge beams are (700 x 350) mm
Columns dimensions are (350x350) mm
Slab thickness is 163 mm



Min (4*163,700-163) = (652,537)

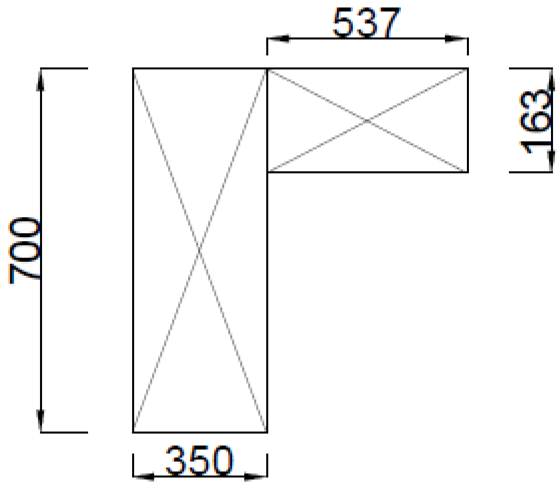
Either the section is divided to:



$$C1=\sum(1-0.63\frac{x}{y})(\frac{x^3y}{3})=[(1-0.63\frac{163}{887})(\frac{163^3*887}{3})+(1-0.63\frac{350}{537})(\frac{350^3*537}{3})]$$

$$C1 = 1.13\times10^9 + 4.54\times10^9 = 5.67\times10^9\ mm^4$$

Or the section is divided to:



$$C2=\sum(1-0.63\frac{x}{y})(\frac{x^3y}{3})=[(1-0.63\frac{350}{700})(\frac{350^3*700}{3})+(1-0.63\frac{163}{537})(\frac{163^3*537}{3})]$$

$$C2 = 6.85\times10^9 + 0.628\times10^9 = 7.48\times10^9\ mm^4$$

The C will be the maximum of the two values:

$$\mathbf{C=7.48\times10^9\ mm^4}$$

Example 1: For the longitudinal interior frame (A) of the flat plate shown in figure below, by using direct design method find:

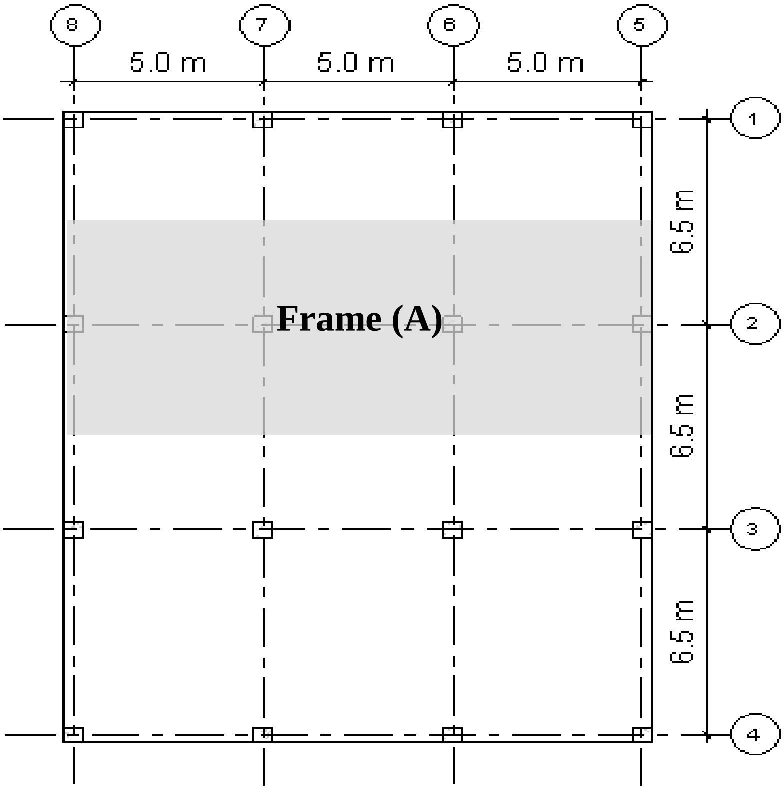
1. Longitudinal distribution of total static moment at factored loads.
2. Lateral distribution of moment at interior and exterior panels.

Slab thickness = 200 mm, d = 180 mm

q_u=15.2 kN/m^2

all columns = 400 x 400 mm

fc` = 25 MPa ,fy=400 MPa



Solution:

1. **Longitudinal distribution of total static moment at factored loads.**

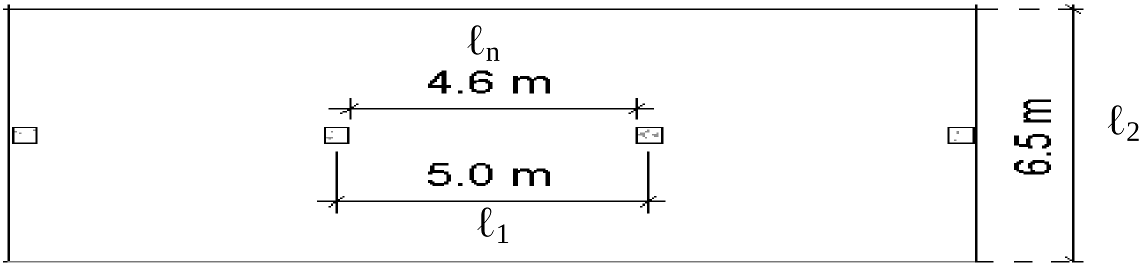
q_u =15.2 kN/m^2

$M_o = \frac{q_u * \ell n^2 * \ell 2}{8}$

$\ell_n = 5-(0.4) = 4.6$ m or $0.65\ell_1 = 0.65*5=3.25$ m use $\boldsymbol{\ell_n = 4.6}$ **m**

$\ell_2 = (\frac{6.5}{2}+\frac{6.5}{2}) = 6.5$ m

$M_o = \frac{15.2*4.6^2*6.5}{8} = 261.33$ kN.m



Longitudinal distribution

For interior span:

Neg. Mu = 0.65 M_o

Pos.Mu = 0.35 M_o

End (exterior) span:

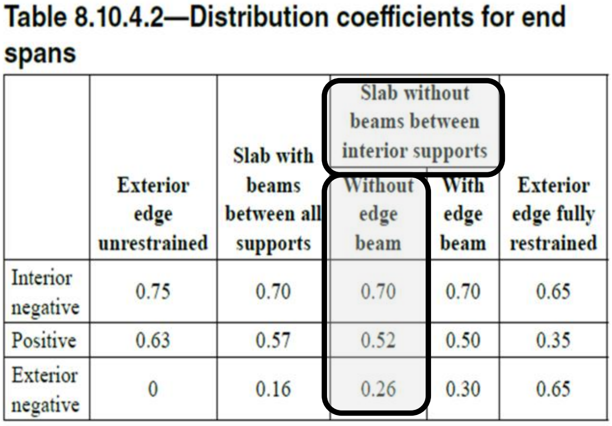
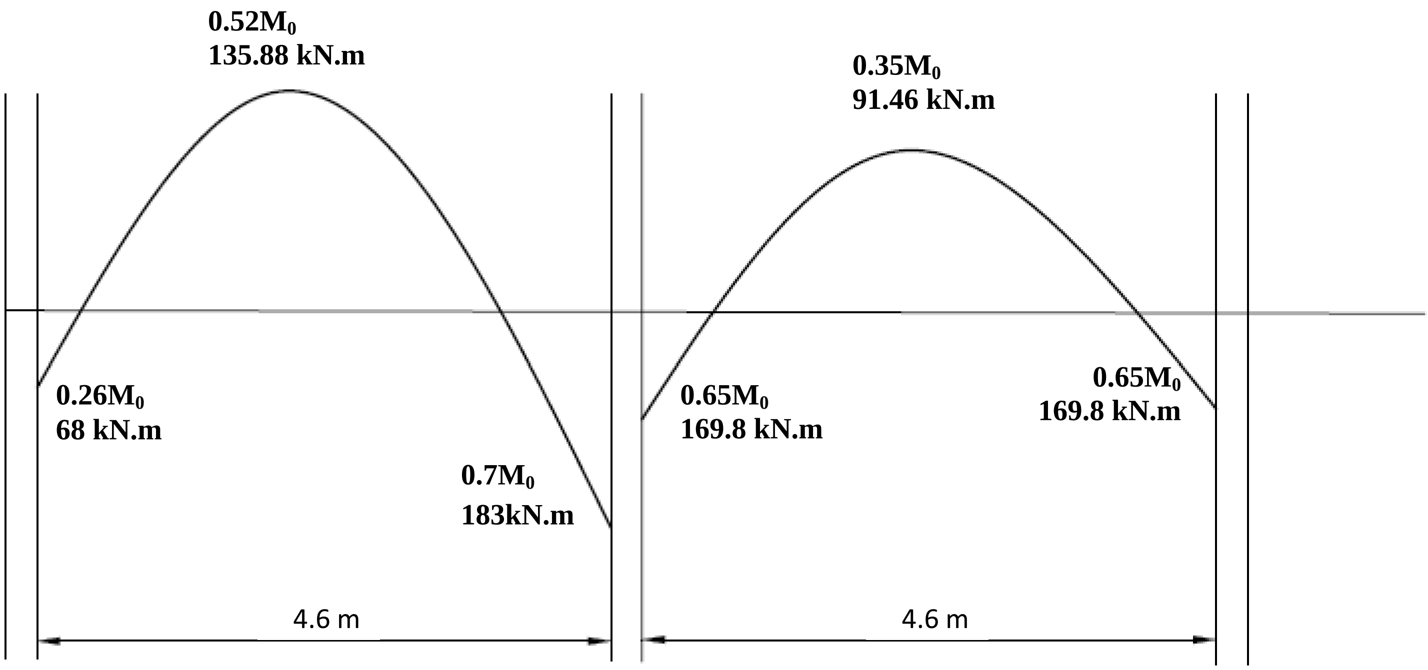


Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



2. Lateral distribution of moment at interior and exterior panels.

For Interior panel

1. Negative Moment

Total Negative moment = 169.8 kN.m

α_{f1} = 0 (no beam in the direction of moment)

$$\frac{\ell_2}{\ell_1} = \frac{6.5}{5} = 1.3$$

C.S%=75%

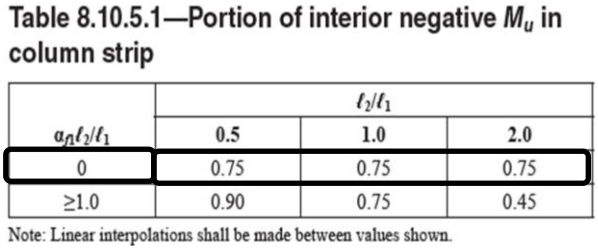


Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Or by using equation:

-Interior C.S $_{\text{coefficient}}$% = 75+30($\alpha_{f1}\frac{\ell_2}{\ell_1}$)* (1- $\frac{\ell_2}{\ell_1}$)

-Interior C.S $_{\text{coefficient}}$% = 75+30*(0)* ($\frac{6.5}{5}$)*(1- $\frac{6.5}{5}$) = 75%

Negative moment at column strip = C.S%*M_{Neg} = 0.75*169.8 = 127.35 kN.m

Negative moment at middle strip = 169.8-127.35 = 42.45 kN.m

2. Positive Moment

Total Positive Moment = 91.46 kN.m

α_{f1} = 0 (no beam in the direction of moment)

$\frac{\ell_2}{\ell_1} = \frac{6.5}{5}$ = 1.3

C.S% = 60%

Or by using equation:

+C.S $_{\text{coefficient}}$% = 60+30($\alpha_{f1}\frac{\ell 2}{\ell 1}$) $\times$ (1.5 $-$ $\frac{\ell 2}{\ell 1}$)

+C.S $_{\text{coefficient}\%}$ = 60+30(0)*($\frac{6.5}{5}$) $\times$ (1.5 $-$ $\frac{6.5}{5}$) = 60%

Positive moment at column strip = 0.6*91.46 = 54.88 kN.m

Positive moment at middle strip = 91.46-54.88 =36.58 kN.m

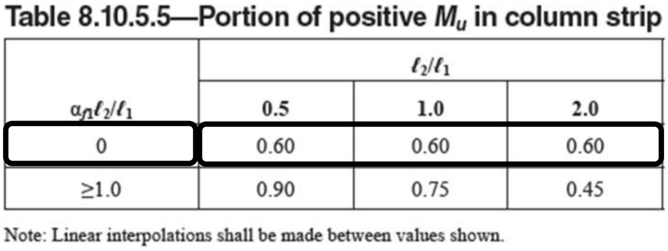


Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

For End (exterior) span

1. Interior Negative moment

Total negative moment =183 kN.m

Negative moment at column strip = 0.75*183 = 137.25 kN.m

Negative moment at middle strip =183-137.25 = 45.75 kN.m

2. Positive moment

Total positive moment = 135.88 kN.m

Positive moment at column strip = 0.6*135.88 = 81.53 kN.m

Positive moment at middle strip = 135.88 -81.53 = 54.35 kN.m

3. Exterior Negative moment

Total negative moment = 68 kN.m

βt = 0 (no edge beam)

α_{f1} = 0 (no beam in the direction of moment)

$\frac{\ell_2}{\ell_1} = \frac{6.5}{5}$ = 1.3

C.S% =100%

Or by using equation

-Exterior C.S $_{\text{coefficient}}$ = 100-10βt +12 βt ($\alpha_{f1}\frac{\ell_2}{\ell_1}$) $\times$ (1$-$ $\frac{\ell_2}{\ell_1}$)

-Exterior C.S $_{\text{coefficient}}$ = 100-10*0+12 $\times$ 0 $\times$ (0) (1$-$ $\frac{6.4}{5}$) =100%

Negative moment at column strip = 1*68 = 68 kN.m

Negative moment at middle strip = 68−68 = 0 ■

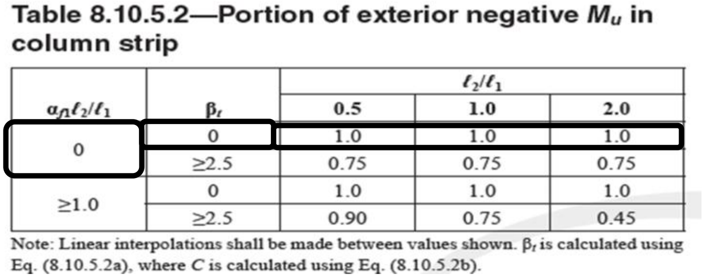


Table 8.10.5.2—Portion of exterior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	β_t	ℓ_2/ℓ_1		
		0.5	1.0	2.0
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

Example 2: for the longitudinal interior frame (A) of the flat plate floor shown in figure below by using direct design method find:

1. Longitudinal distribution of the static moment at factored loads.
2. Lateral distribution of the moment at exterior support.

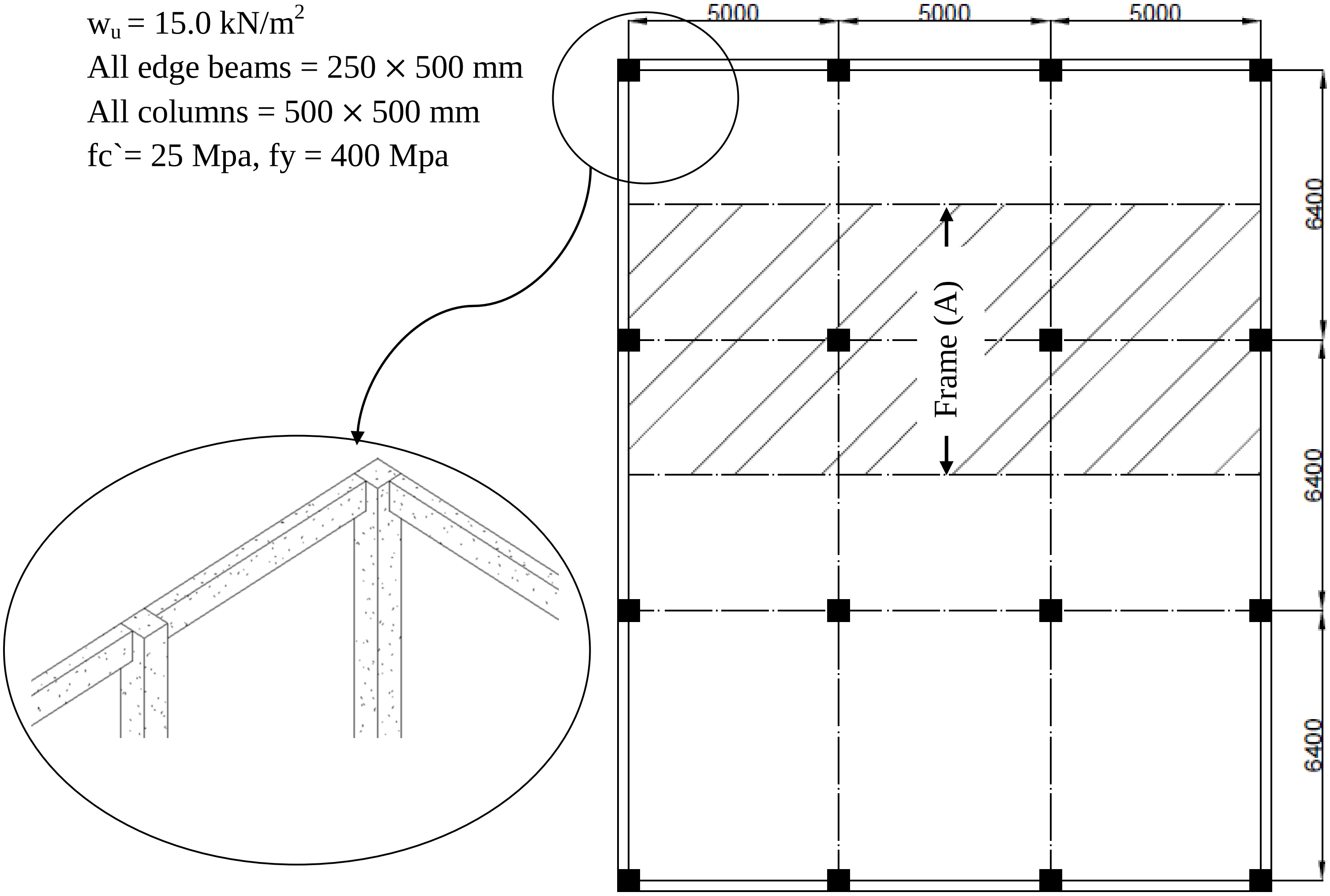
Slab thickness = 200 mm, d =165 mm

$w_u = 15.0$ kN/m^2

All edge beams = 250 × 500 mm

All columns = 500 × 500 mm

fc\`= 25 Mpa, fy = 400 Mpa



Solution:

1. Longitudinal distribution of the static moment at factored loads.

For frame (A)

$\ell_1 = 5000$ mm

$\ell_n = 5000 - 500 = 4500$ mm or $0.65\ \ell_1$

$\ell_n =$ **4500** mm or 3250 mm use larger

$\ell_2 = \left(\frac{6400}{2} + \frac{6400}{2}\right) = 6400$ mm

$$M_o = \frac{Wu*\ell n^2*\ell 2}{8}$$

$$M_o = \frac{15*4.5^2*6.4}{8} = 243 \text{ kN.m}$$

Longitudinal distribution of total static moment at factored loads

Interior span:

Neg. Mu = $\mathbf{0.65M_o}$=0.65*243 = 157.95 kN.m

Pos.Mu = $\mathbf{0.35M_o}$=0.35*243 = 85.05kN.m

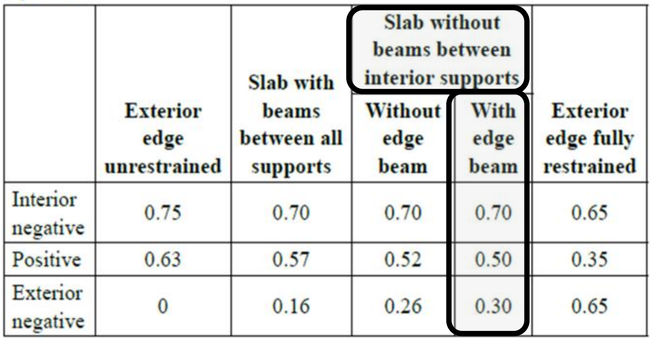
End Span:

Interior Neg.Mu = $\mathbf{0.7M_o}$=0.7*243 = 170.1 kN.m

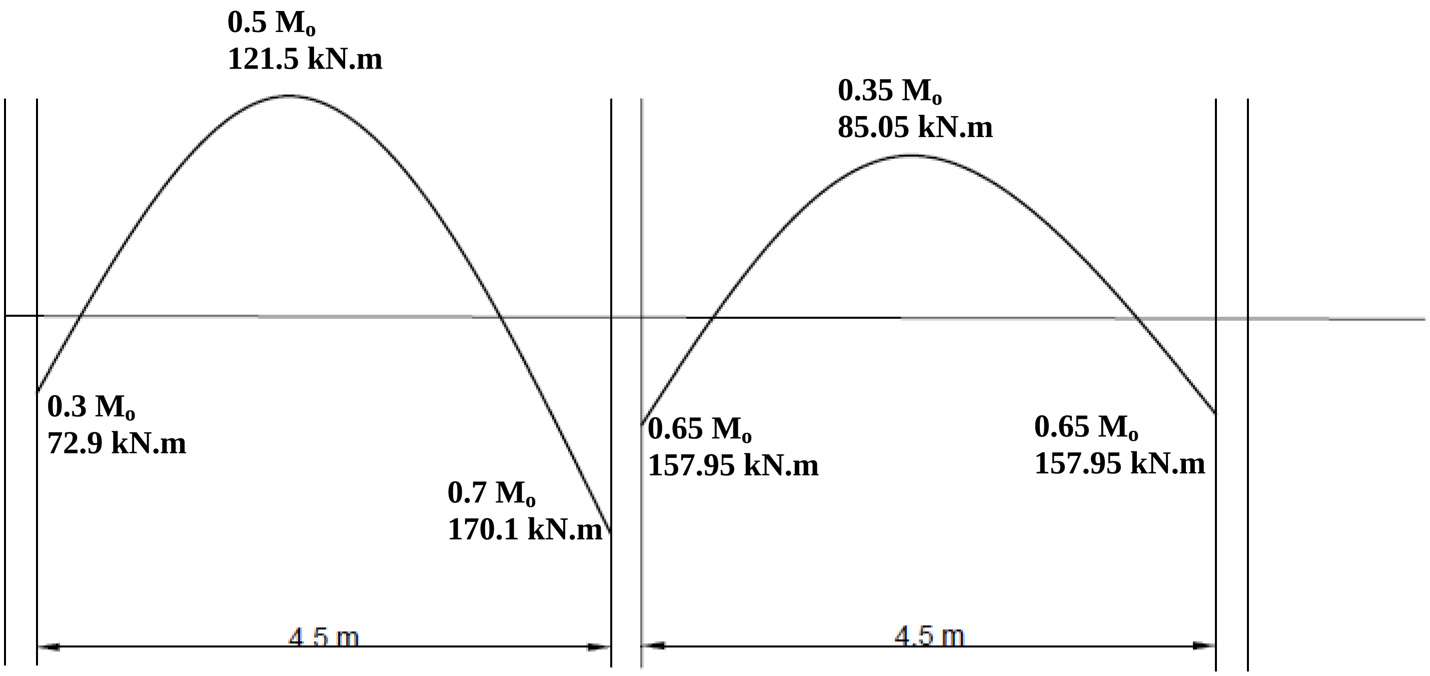
Pos.Mu=$\mathbf{0.5M_o}$ = 0.5*243 = 121.5 kN.m

Exterior Neg.Mu = $\mathbf{0.3M_o}$=0.3*243 = 72.9 kN.m

Table 8.10.4.2—Distribution coefficients for end spans



	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



2. Lateral distribution of the moment at exterior support.

Negative moment at exterior support = 72.9 kN.m

Negative $M_{C.S}$ Exterior = C.S $_{coefficient}$%*-M Exterior

Negative $M_{M.S}$ = M - Negative $M_{C.S}$

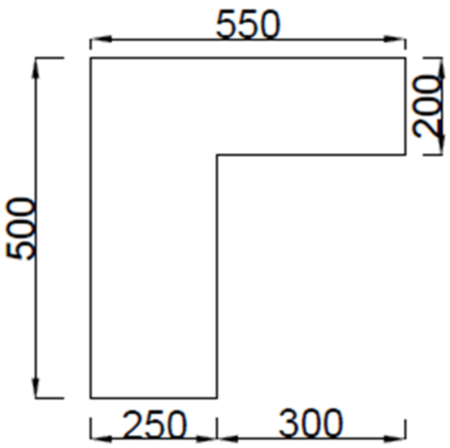
-Exterior C.S $_{coefficient}$% = 100-10βt + 12 βt $(\alpha_{f1}\frac{\ell_2}{\ell_1})$* $(1-\frac{\ell_2}{\ell_1})$

α_{f1} = 0 (no beam in the direction of moment)

Find βt:

$$\beta_t = \frac{C}{2I_S}$$

Calculate C:



$C = \sum(1 - 0.63\frac{x}{y})(\frac{x^3 y}{3})$

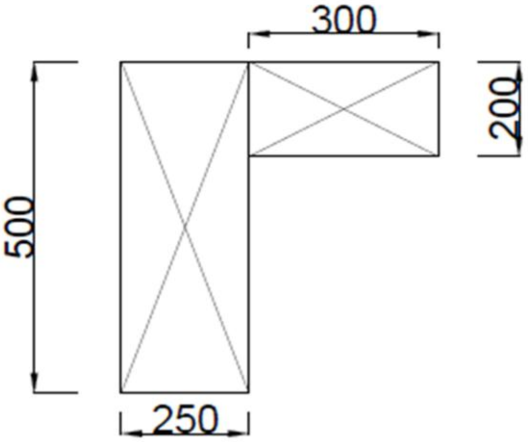
$C1 = (1 - 0.63 * \frac{200}{300})(\frac{200^3 * 300}{3}) + (1 - 0.63\frac{250}{500})(\frac{250^3 * 500}{3})$

$C1 = 4.64*10^8 + 1.783*10^9 = 2.2478*10^9$ mm^4

$C2 = (1 - 0.63 * \frac{200}{550})(\frac{200^3 * 550}{3}) + (1 - 0.63 * \frac{250}{300})(\frac{250^3 * 300}{3})$

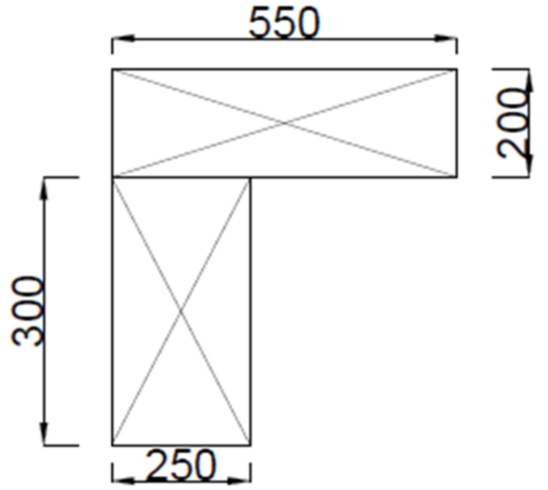
$C2 = 1.13*10^9 + 7.42*10^8 = 1.8721*10^9$ mm^4

$\therefore C = 2.2478*10^9$ mm^4 **(larger value)**



$Is = \frac{\ell_2 * h^3}{12} = \frac{6400*200^3}{12} = 4.267*10^9$ mm^4

$\beta_t = \frac{C}{2I_S} = \frac{2.2478*10^9}{2*4.267*10^9} = 0.263$



-Exterior C.S coefficient% = $100 - 10\beta t + 12\,\beta t\,(\alpha_{f1}\frac{\ell_2}{\ell_1}) \times (1 - \frac{\ell_2}{\ell_1})$

$= 100 - 10 \times 0.263 + 12 \times 0.263 \times (0*\frac{6.4}{5})\,(1 - \frac{6.4}{5})$

-Exterior C.S coefficient% = 97.37%

Negative moment at column strip for exterior support = 0.9736×72.9 = 70.975 kN.m
Negative moment at middle strip for exterior support = 72.9-70.975 =1.925 kN.m ■

Example 3: For the longitudinal interior frame of the flat plate shown in figure, by using the direct design method, Find:

a. Longitudinal distribution of total static moment at factored loads.

b. Lateral distribution of moment at exterior panel.

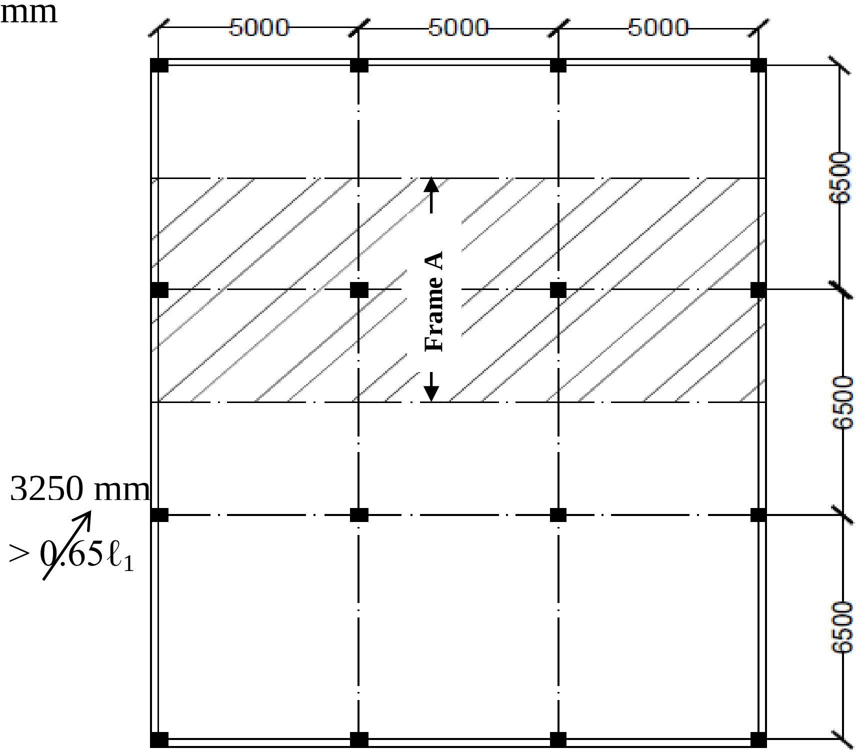
Slab thickness = 180 mm, d =150 mm

Wu=14 kN/m^2

All edge beams = 250 x 500 mm

All columns = 400 x 400 mm

fc`=24 Mpa , fy=400 Mpa



Solution:

a.

For frame (A)

$\ell_1 = 5000$ mm

$\ell_2 = 6500$ mm

$\ell_n = \ell_1 - 400 = 5000 - 400 = 4600$ mm $> 0.65\ell_1$

$$M_o = \frac{Wu*\ell n^2*\ell 2}{8}$$

$$M_o = \frac{14*4.6^2*6.5}{8} = 240.695 \text{ kN.m}$$

Longitudinal distribution of total static moment at factored loads

Interior span:

Neg.M = 0.65M_o

Pos.M = 0.35M_o

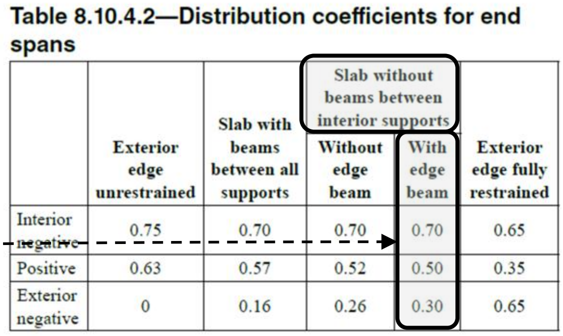
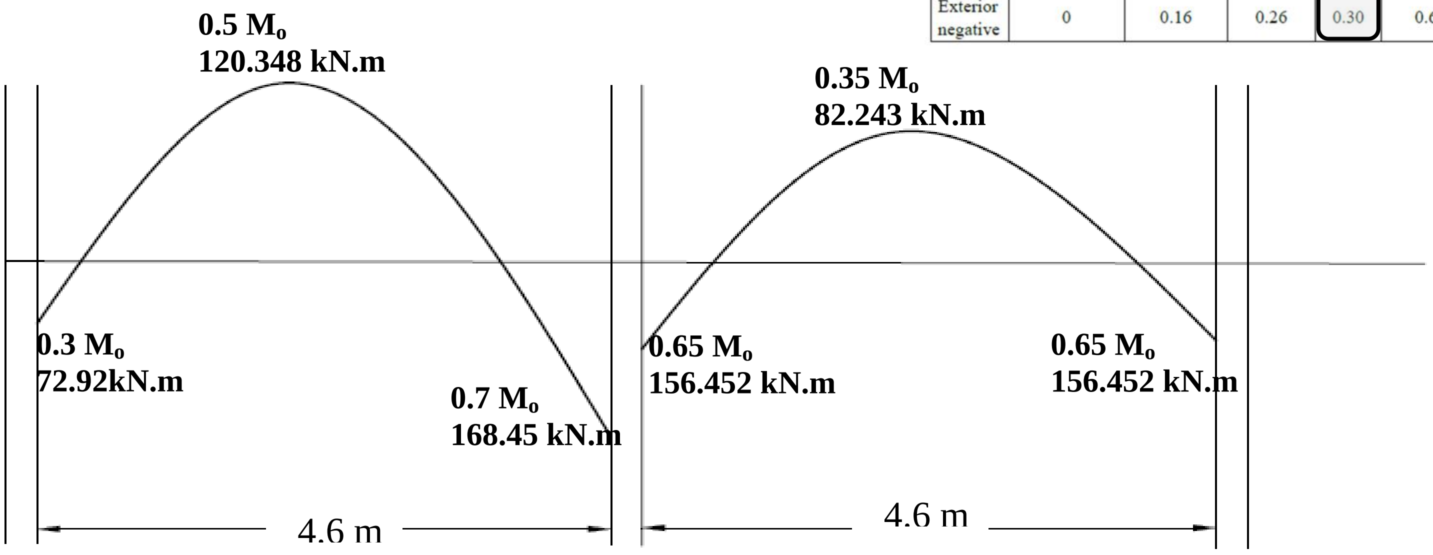


Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

End span:



b.

Exterior Panel

1. Negative moment at interior support

Total negative moment = 168.45 kN.m

α_{f1} = 0 (no beam in the direction of moment)

$\frac{\ell 2}{\ell 1} = \frac{6.5}{5}$ = 1.3

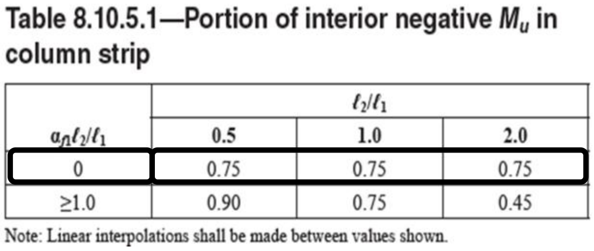


Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

-Interior C.S coefficient % = 75 + $30(\cancelto{0}{\alpha_{f1}\frac{\ell_2}{\ell_1}})$* (1- $\frac{\ell_2}{\ell_1}$)

-Interior C.S coefficient % = 75 + 30($0 * 1.3$)* (1- 1.3) = 75%

Negative moment at column strip = 0.75*168.45 = 126.34 kN.m

Negative moment at middle strip = 168.45-126.34 = 42.11 kN.m

2. Positive Moment

Total positive moment =120.348 kN.m

α_{f1} = 0 (no beam in the direction of moment)

$\frac{\ell 2}{\ell 1} = \frac{6.5}{5}$ =1.3

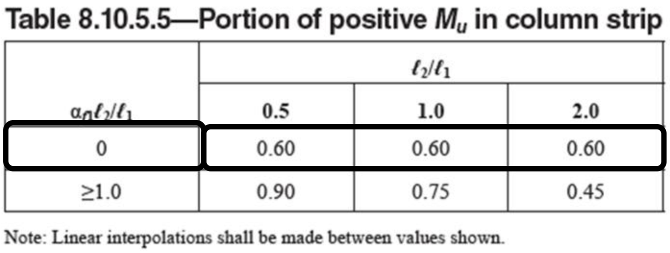


Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

+C.S coefficient% = 60 + 30($\alpha_{f1}\frac{\ell_2}{\ell_1}$)* (1.5- $\frac{\ell_2}{\ell_1}$)

+C.S coefficient% = 60 + 30(0 $*$ 1.3)* (1.5- 1.3) = 60%

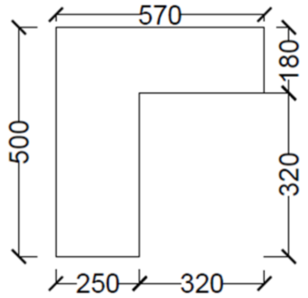
Positive moment at column strip = 0.6*120.348 = 72.21 kN.m

Positive moment at middle strip = 120.348−72.21 = 48.138 kN.m

3. Negative moment at exterior support

Total negative moment = 72.209

α_{f1} = 0 (no beam in the direction of moment)



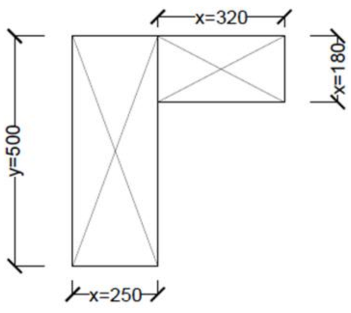
-Exterior C.S coefficient% =100-10βt +12 βt ($\alpha_{f1}\frac{\ell 2}{\ell 1}$)* (1- $\frac{\ell 2}{\ell 1}$)

Find βt:

$$\beta t = \frac{C}{2I_S}$$

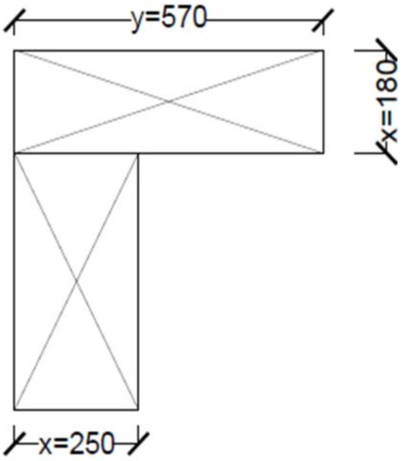
Calculate C:

$$C = \sum(1 - 0.63\frac{x}{y})(\frac{x^3 y}{3})$$



C1=$(1 - 0.63 * \frac{180}{320})(\frac{180^3 * 320}{3})$+ $(1 - 0.63\frac{250}{500})(\frac{250^3 * 500}{3})$

C1=4.016*10^8+1.783*10^9=2.185*10^9 mm^4



C2 = $(1-0.63*\frac{180}{570})(\frac{180^3*570}{3})$+ $(1-0.63\frac{250}{320})(\frac{250^3*320}{3})$

C2 = $8.87*10^8+8.46*10^8=1.733*10^9$ mm^4

$\therefore$ C =$2.185*10^9$ mm^4 **(larger value)**

Is = $\frac{\ell_2*h^3}{12}=\frac{6500*180^3}{12}$ = $3.159*10^9$ mm^4

Bt = $\frac{C}{2I_S}=\frac{2.185*10^9}{2*3.159*10^9}=0.346$

-Exterior C.S $_{\text{coefficient}}$% =100-10βt + 12 βt $(\alpha_{f1}\frac{\ell 2}{\ell 1})\times(1-\frac{\ell 2}{\ell 1})$

=100 - 10 × 0.346 + 12 × 0.346 × (0*1.3) (11.3)

-Exterior C.S $_{\text{coefficient}}$%=96.54%

Negative moment at column strip = 0.9654*72.209 = 69.71 kN.m

Negative moment at middle strip = 72.209-69.71= 2.499 kN.m ■

Example 4: for the longitudinal interior (frame A) of the flat plate floor show in figure, by using the direct design method finds:

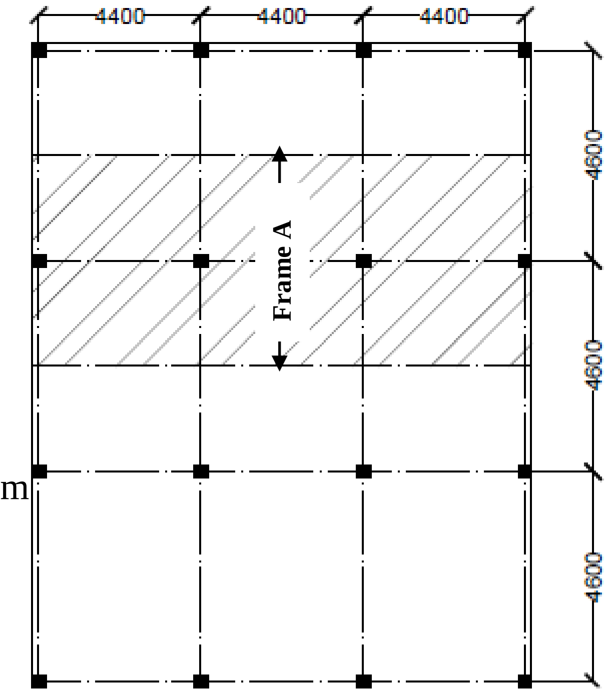
a. Longitudinal distribution of the total static moment at factored loads.
b. Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).

Slab thickness = 180 mm, d =150 mm

q_u =15 kN/m^2

All columns = 400 x 400 mm

fc`=24 Mpa , fy=400 Mpa



Solution:

a.

For frame (A)

$\ell_1 = 4400$ mm

$\ell_2 = 4600$ mm

$\ell_n = \ell_1$-400=4400-400=4000 mm > $0.65\ell_1$=2860 mm

$$M_o = \frac{Wu * \ell_n^2 * \ell_2}{8}$$

$$M_o = \frac{15*4^2*4.6}{8} = 138 \text{ kN.m}$$

Longitudinal distribution of total static moment at factored loads

Interior span:

Neg.M = $0.65M_o$

Pos.M = $0.35M_o$

End span:

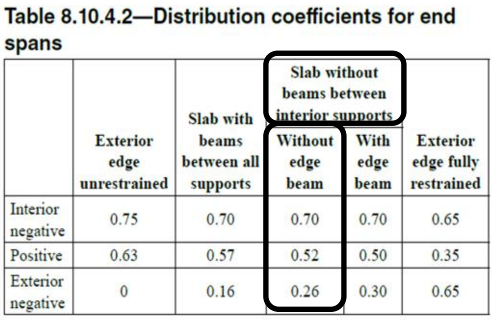
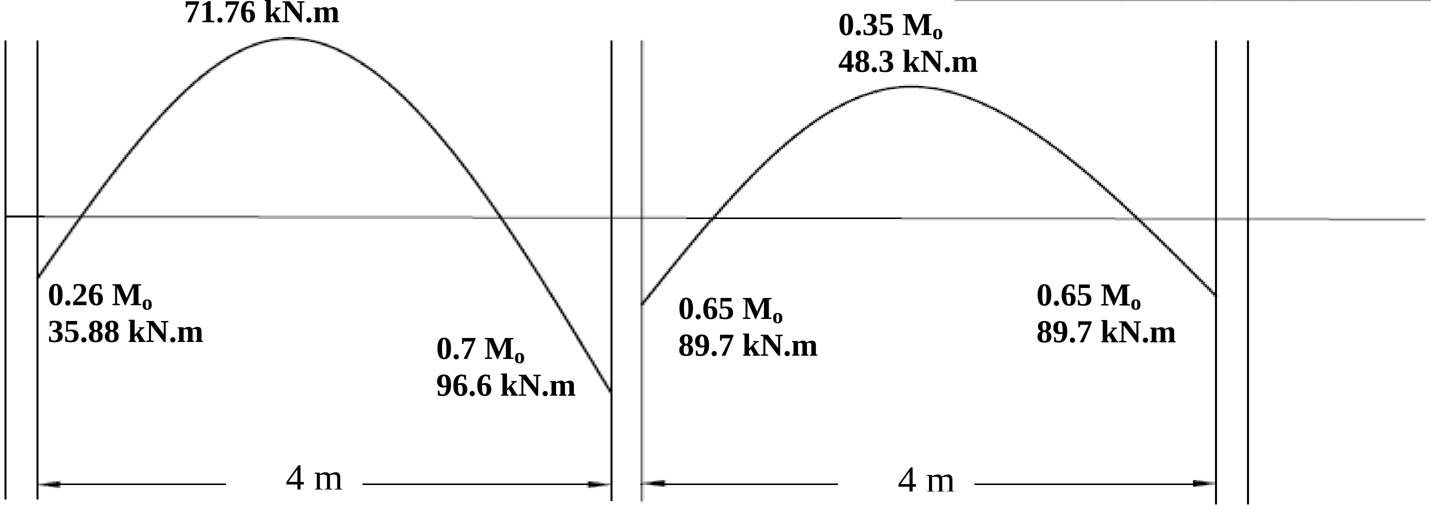


Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



b.

Interior Panel

1. **Negative moment**

Total negative moment = 89.7 kN.m

α_{f1} = 0 (no beam in the direction of moment)

-Interior C.S $_{\text{coefficient}}$ % = $75+30(\alpha_{f1}\frac{\ell_2}{\ell_1})*(1-\frac{\ell_2}{\ell_1})$

-Interior C.S $_{\text{coefficient}}$ % =75%

Negative moment at column strip = 0.75*89.7 = 67.275 kN.m

Negative moment at middle strip = 89.7-67.275 = 22.425 kN.m

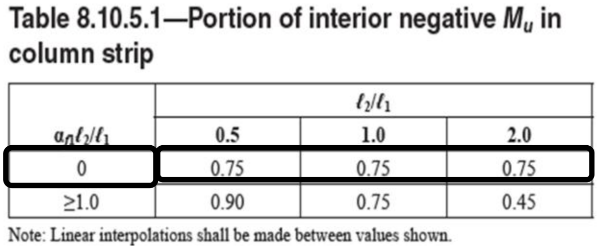


Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	**1.0**	**2.0**
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

2. **Positive moment**

Total positive moment = 48.3 kN.m

α_{f1} = 0 (no beam in the direction of moment)

+C.S $_{\text{coefficient}}$%=$60+30(\alpha_{f1}\frac{\ell_2}{\ell_1})*(1.5-\frac{\ell_2}{\ell_1})$

+C.S $_{\text{coefficient}}$% = 60%

Positive moment at column strip = 0.6*48.3 = 28.98 kN.m

Positive moment at middle strip = 48.3 -28.98 =19.32 kN.m ■

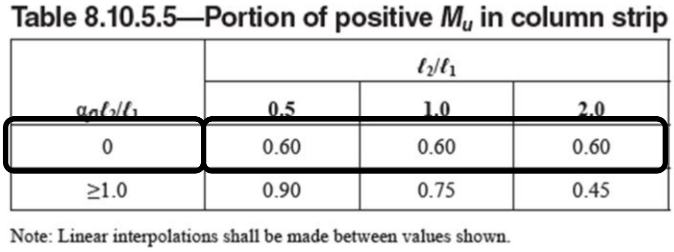


Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	**1.0**	**2.0**
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Example 5: For the exterior longitudinal frame (Frame B) of the flat late floor shown in figure, and by using the Direct Design method, Find:

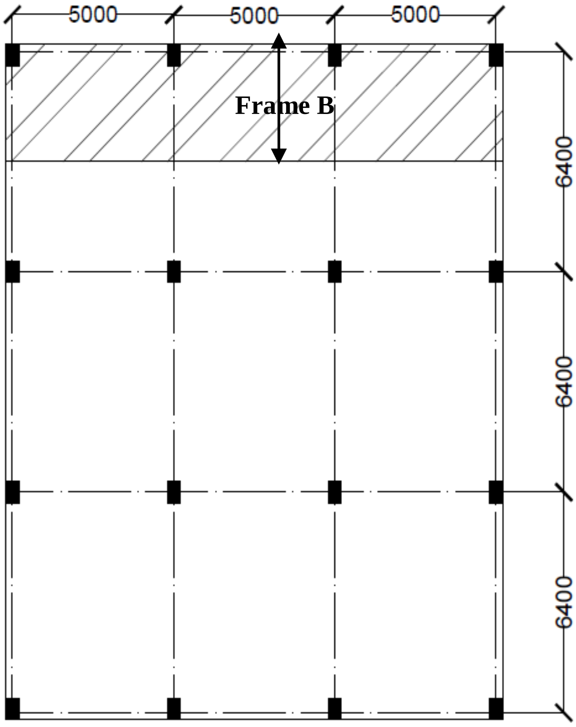
a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of moment at exterior panel

Slab thickness = 175 mm, d =140 mm

W_u =14 kN/m^2

All columns = 600 x 400 mm



Solution:

a.

For frame (b)

$\ell_1 = 5000$ mm

$\ell_2 = \frac{6400}{2} + \frac{600}{2} = 3200 + 300 = 3500$ mm

$\ell_n = \ell_1 - 400 = 5000 - 400 = 4600$ mm $> 0.65\ell_1 = 3250$ mm

$$M_o = \frac{Wu * \ell n^2 * \ell 2}{8}$$

$$M_o = \frac{14 * 4.6^2 * 3.5}{8} = 129.6 \text{ kN.m}$$

Longitudinal distribution of total static moment at factored loads

Interior span:

Neg.M = 0.65M_o

Pos.M = 0.35M_o

End span:

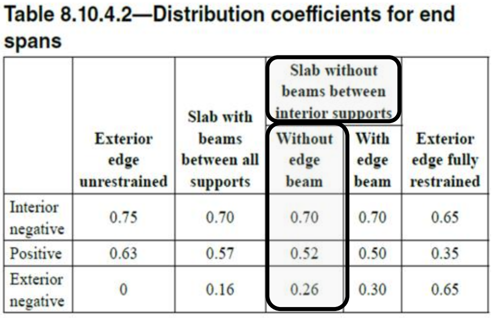
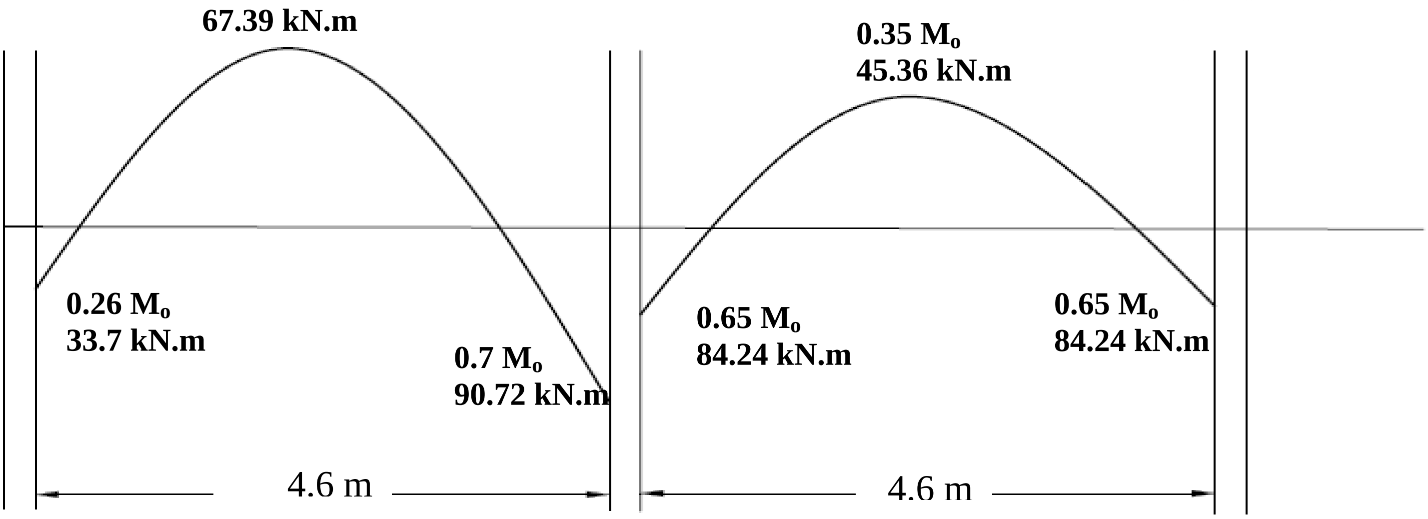


Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



b.

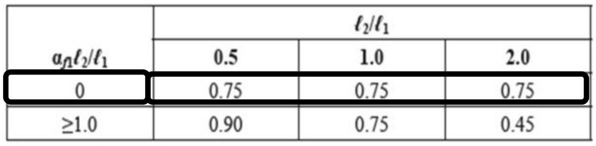
Exterior Panel

1. Interior Negative moment

Total negative moment = 90.72 kN.m

α_{f1} = 0 (no beam in the direction of moment)

Table 8.10.5.1—Portion of interior negative M_u in column strip



	ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

-Interior C.S coefficient $\% = 75+30(\alpha_{f1}\frac{\ell 2}{\ell 1})*(1-\frac{\ell 2}{\ell 1})$

-Interior C.S coefficient %=75%

Negative moment at column strip=0.75*90.72 =68.04 kN.m

Negative moment at middle strip=90.72 -68.04 =22.68 kN.m

2. Positive moment

Total positive moment = 67.39 kN.m

α_{f1} = 0 (no beam in the direction of moment)

Table 8.10.5.5—Portion of positive M_u in column strip

	ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

+C.S coefficient$\% = 60 + 30(\alpha_{f1}\frac{\ell 2}{\ell 1})*(1.5-\frac{\ell 2}{\ell 1})$

+C.S coefficient% = 60%

Positive moment at column strip = 0.6*67.39 =40.434 kN.m

Positive moment at middle strip = 67.39 − 40.434 =27.497 kN.m

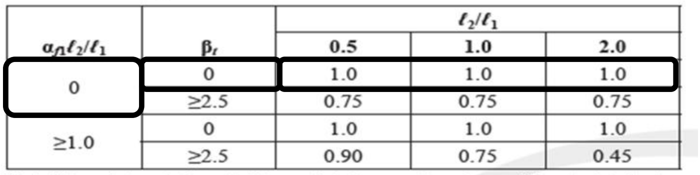
3. Exterior Negative Moment

Total negative moment = 33.7 kN.m

α_{f1} =0 (no beam in the direction of moment)

βt = 0 (no Edge Beam)

Table 8.10.5.2—Portion of exterior negative M_u in column strip



		ℓ_2/ℓ_1		
$\alpha_{f1}\ell_2/\ell_1$	β_t	0.5	1.0	2.0
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

-Exterior C.S coefficient$\% = 100 - 10\beta t + 12\ \beta t\ (\alpha_{f1}\frac{\ell 2}{\ell 1})*(1-\frac{\ell 2}{\ell 1})$

-Exterior C.S coefficient% = 100%

Negative moment at column strip =1*33.7 = 33.7 kN.m

Negative moment at middle strip = 33.7-33.7= 0 kN.m ■

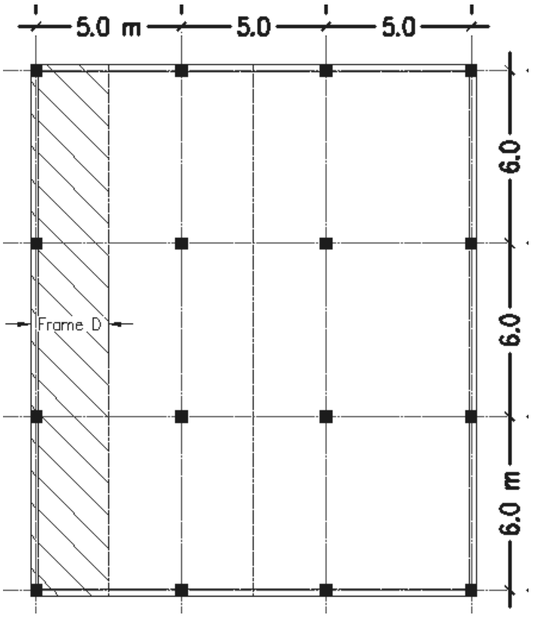
Example 6: For the the transverse extiror frame (Frame D) of the falt plate floor, without edge beams, shown in Figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).

Slab thickness = 180 mm, d = 150 mm

$q_u = 15.0\ kN/m^2$

All columns = 400×400 mm



Solution

a.

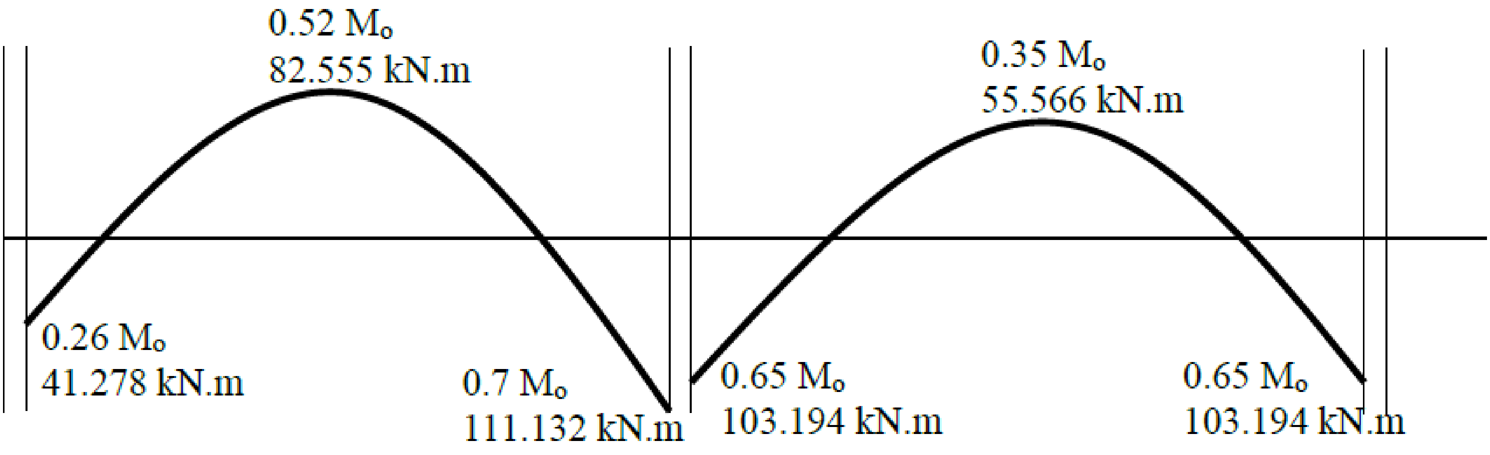
$\ell_1 = 6.0\ m$

$\ell_2 = 2.7\ m$

$\ell_n = \ell_1 - 0.4 = 6.0 - 0.4 = 5.6\ m$

$$M_o = \frac{qu * \ell n^2 * \ell 2}{8}$$

$$M_o = \frac{15 * 5.6^2 * 2.7}{8} = 158.76\ kN.m$$



Longitudinal distribution of total static moment at factored loads

b.

Interior panel

1. Negative moment

Total moment = 103.194 kN.m

Negative moment at column strip = 103.194 × 0.75 = 77.396 kN.m

Negative moment at middle strip = 103.194 - 77.396 = 25.798 kN.m

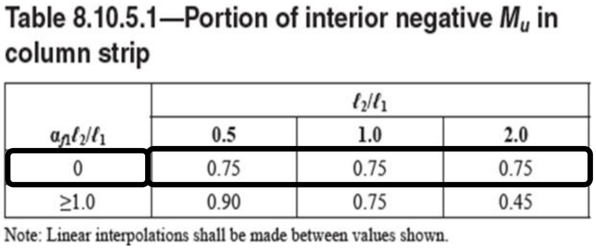


Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

2. Positive moment

Total moment = 55.566 kN.m

Negative moment at column strip = 55.566 × 0.60 = 33.34 kN.m

Negative moment at middle strip = 55.566 - 33.34 = 22.226 kN.m

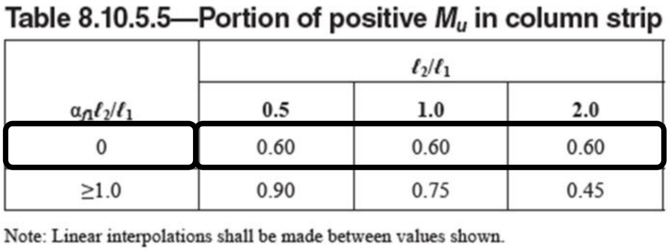


Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

Example 7: For the transverse interior frame (Frame A) of the flat plate floor (without edge beams) shown in Figure. By using the Direct Design Method, find:

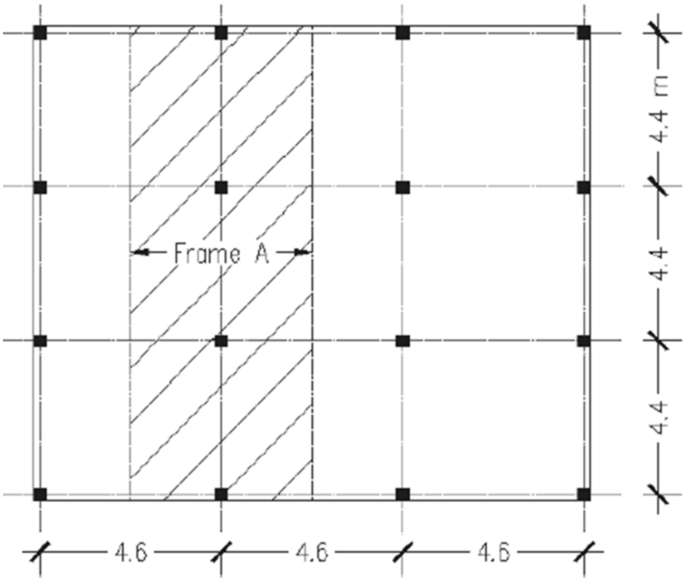
a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).

Slab thickness = 180 mm, d = 150 mm

$q_u = 15.0 \text{ kN/m}^2$

All columns = 400×400 mm



Solution

a.

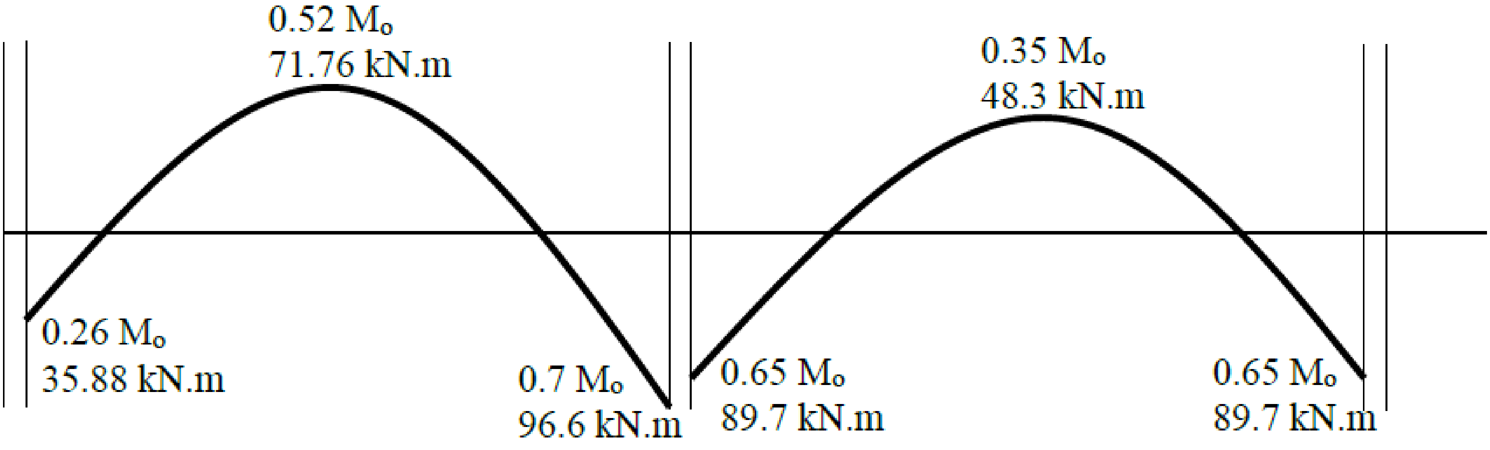
$\ell_1 = 4.4$ m

$\ell_2 = 4.6$ m

$\ell_n = \ell_1 - 0.4 = 4.4 - 0.4 = 4.0$ m

$$M_o = \frac{qu*\ell n^2*\ell 2}{8}$$

$$M_o = \frac{15*4^2*4.6}{8} = 138.0 \text{ kN.m}$$



Longitudinal distribution of total static moment at factored loads

b.

Interior panel

1. Negative moment

Total moment = 89.7 kN.m

Negative moment at column strip = 89.7×0.75 = 67.275 kN.m

Negative moment at middle strip = 89.7-67.275 = 22.425 kN.m

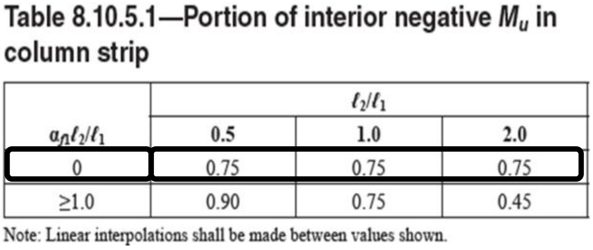


Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

2. Positive moment

Total moment = 48.3 kN.m

Negative moment at column strip = 48.3×0.60 = 28.98 kN.m

Negative moment at middle strip = 48.3-28.98 = 19.32 kN.m

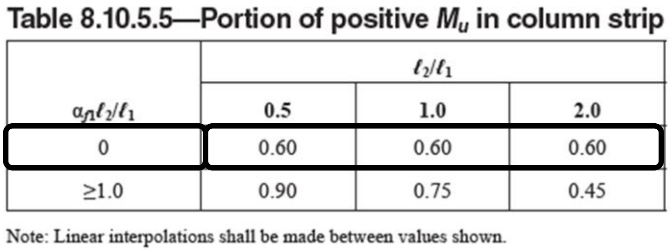


Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

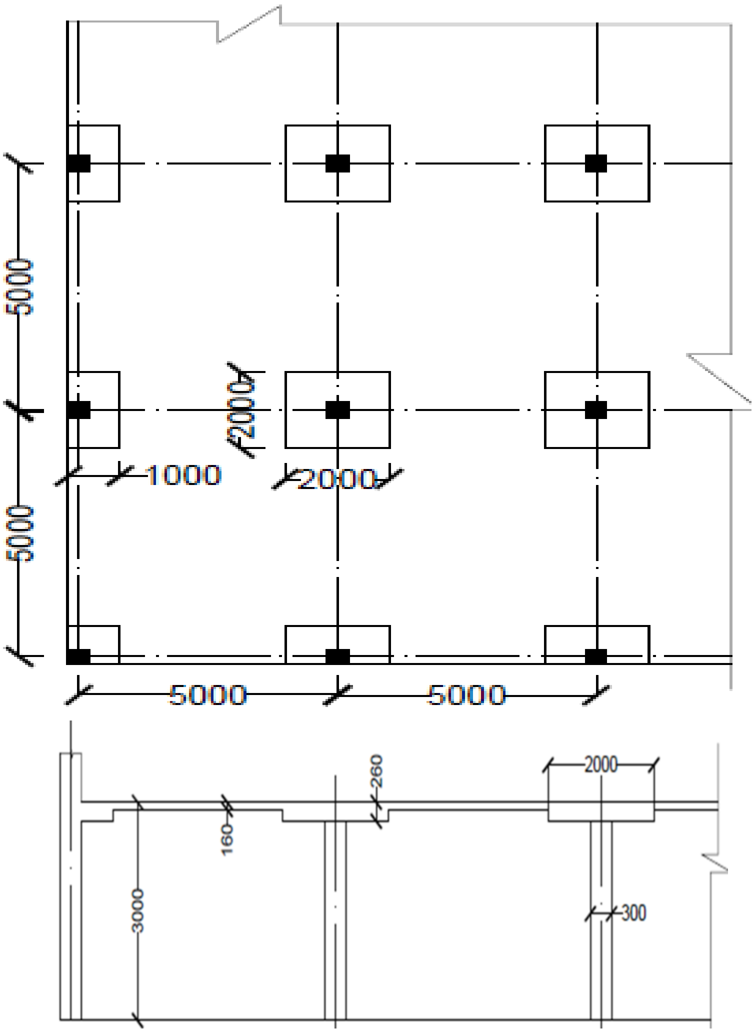
Example 8: For the exterior transverse frame of the flat slab shown in Figure, by using direct design method find:

a. Longitudinal distribution of total static moment at factored loads.

b. Lateral distribution of moment at exterior panel.

D = 6.5 kN/m^2 (include self weight)

L = 5.0 kN/m^2



Solution

a.

$\ell_1 = 5$ m

$\ell_2 = \frac{5}{2} + \frac{0.3}{2} = 2.65$ m

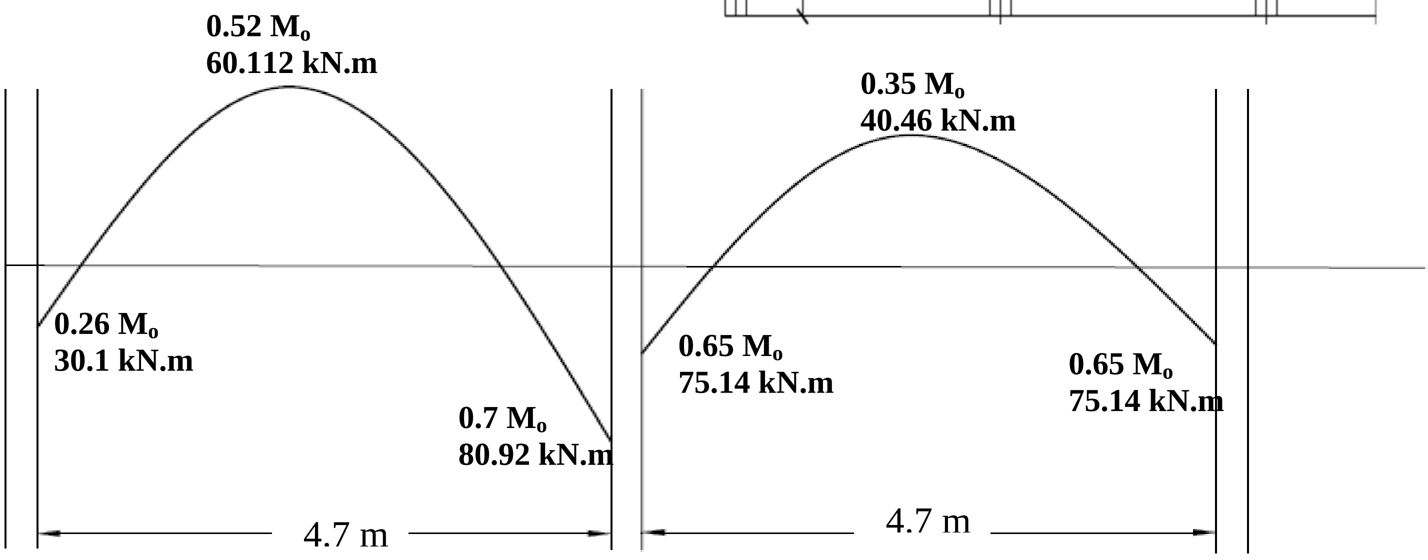
$\ell_n = \ell_1 - 0.3 = 5 - 0.3 = 4.7$ m

W_u=1.2 W_D+1.6W_L

W_u=1.2 *6.5+1.6*5 =15.8 kN/m^2

$M_o = \frac{wu*\ell n^2*\ell 2}{8}$

$M_o = \frac{15.8*4.7^2*2.65}{8}$ =115.6 kN.m

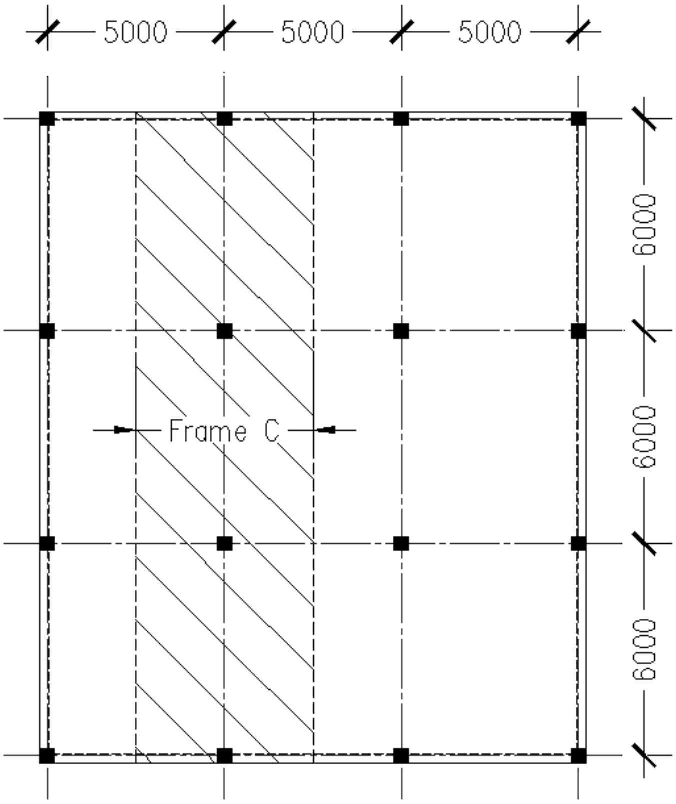


c. Solve it as **homework**.

Home Work:

For the the transverse interior frame (Frame C) of the flat plate floor with edge beams shown in Figure, by using the Direct Design Method, find:

1. Longitudinal distribution of total static moment at factored loads.
2. Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).
3. Lateral distribution of moment at exterior panel (column and middle strip moments at negative and positive moments).



Slab thickness = 180 mm, d = 150 mm

$q_u = 16.0\,kN/m^2$

All edge beams = 250 × 500 mm

All columns = 500 × 500 mm

Home Work Solution:

For the the transverse interior frame (Frame C) of the flat plate floor with edge beams shown in Figure, by using the Direct Design Method, find:

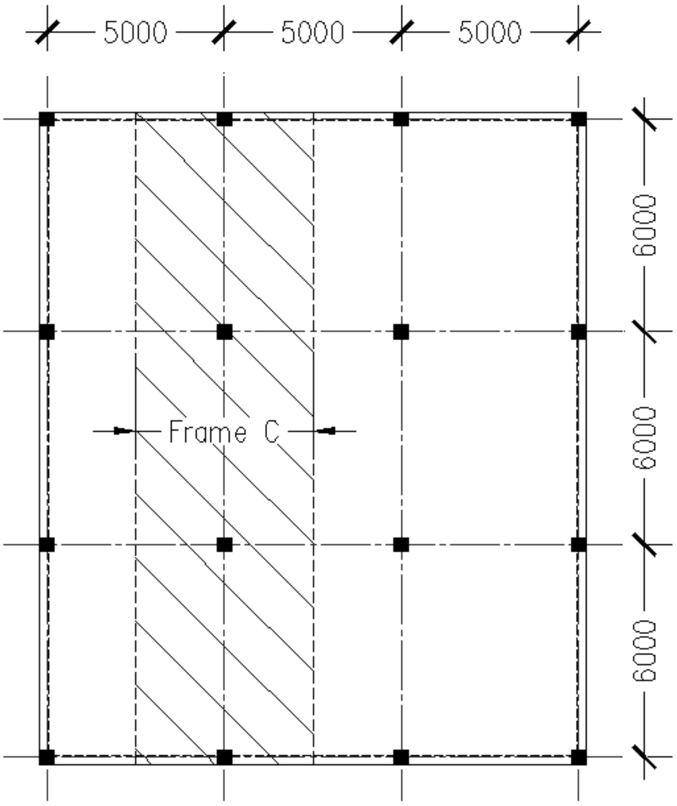
1. Longitudinal distribution of total static moment at factored loads.
2. Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).
3. Lateral distribution of moment at exterior panel (column and middle strip moments at negative and positive moments).

$Slab\ thickness = 180\ mm,\ d = 150\ mm$

$q_u = 16.0\,kN/m^2$

$All\ edge\ beams = 250 \times 500\ mm$

$All\ columns = 500 \times 500\ mm$



Solution

$\ell_n = 6 - 0.5 = 5.5\text{m} > 0.65 \times 6 = 3.9\text{ m}$

$\ell_2 = 5\text{ m}$

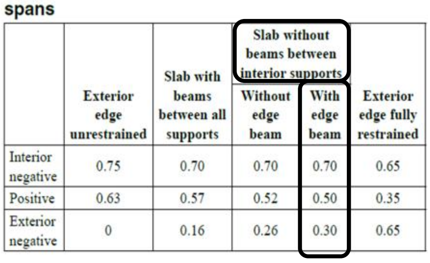
$q_u = 16\text{ kN/m}^2$

$$M_o = \frac{W_u \ell_2 \ell_n^2}{8}$$

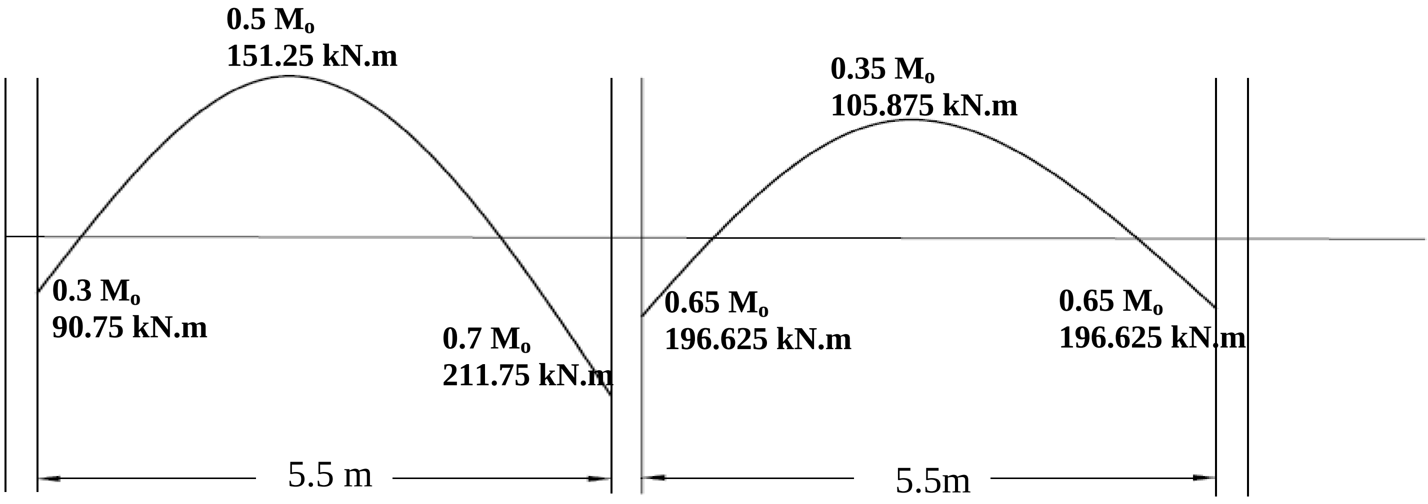
$$M_o = \frac{16 \times 5.5^2 \times 5}{8} = 302.5\text{ kN.m}$$

1. Longitudinal distribution of total static moment at factored loads.

Table 8.10.4.2—Distribution coefficients for end spans



	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



2. Lateral distribution of moment at ***interior panel*** (column and middle strip moments at negative and positive moments).

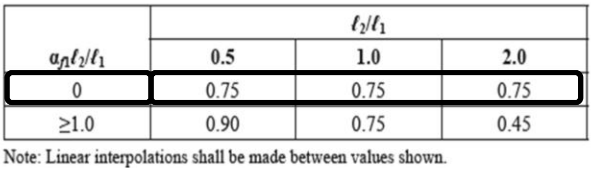
Negative moment

Total moment = 196.625 kN.m

Negative moment at column strip = 196.625 × 0.75 = 147.5 kN.m

Negative moment at middle strip = 196.625−147.5 = 49.125 kN.m

Table 8.10.5.1—Portion of interior negative M_u in column strip



$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

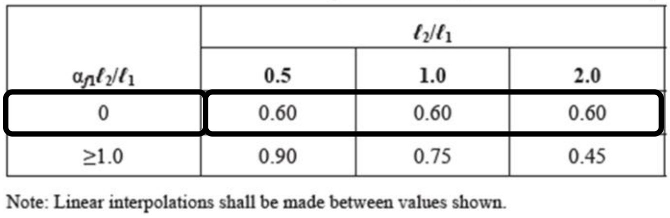
Positive moment

Total moment = 105.875 kN.m

Negative moment at column strip = 105.875 × 0.60 = 63.525 kN.m

Negative moment at middle strip = 105.875 − 63.525 = 42.35 kN.m

Table 8.10.5.5—Portion of positive M_u in column strip



$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	1.0	2.0
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

3. Lateral distribution of moment at ***exterior panel*** (column and middle strip moments at negative and positive moments).

Negative moment at interior support

Total negative moment = 211.75 kN.m

Negative moment at column strip = 211.75 × 0.75 = 158.8125 kN.m

Negative moment at middle strip = 211.75 −158.8125 = 52.94 kN.m

Positive moment

Total moment = 151.25 kN.m

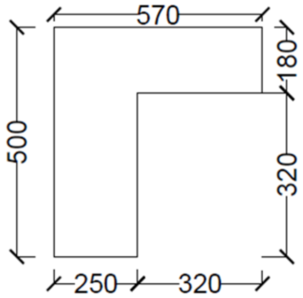
Negative moment at column strip = 151.25 × 0.60 = 90.75 kN.m

Negative moment at middle strip = 151.25 – 90.75 = 60.5 kN.m

Negative moment at exterior support

Total negative moment = 90.75 kN.m

α_{f1} = 0 **(no beam in the direction of moment)**



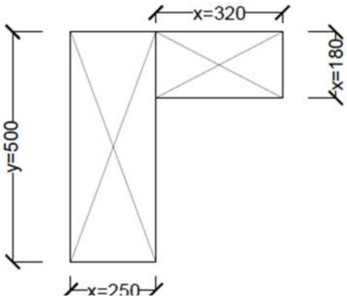
-Exterior C.S coefficient% $=100-10\beta t+12\,\beta t\,(\alpha_{f1}\frac{\ell 2}{\ell 1})*(1-\frac{\ell 2}{\ell 1})$

Find βt:

$$\beta_t=\frac{C}{2I_S}$$

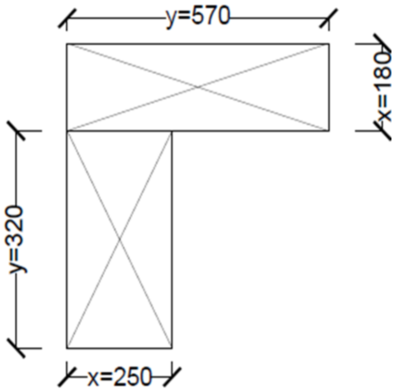
Calculate C:

$$C=\sum(1-0.63\frac{x}{y})(\frac{x^3y}{3})$$



$$C1=\left(1-0.63\times\frac{180}{320}\right)\left(\frac{180^3\times320}{3}\right)+\left(1-0.63\times\frac{250}{500}\right)\left(\frac{250^3\times500}{3}\right)$$

$C1 = 4.016\times10^8 + 1.783\times10^9 = 2.185\times10^9\ mm^4$



$$C2=\left(1-0.63*\frac{180}{570}\right)\left(\frac{180^3*570}{3}\right)+(1-0.63\frac{250}{320})(\frac{250^3*320}{3})$$

$C2 = 8.87\times10^8 + 8.46\times10^8 = 1.733*10^9\ mm^4$

∴ $C = 2.185\times10^9\ mm^4$ **(larger value)**

$$Is=\frac{\ell_2*h^3}{12}=\frac{5000*180^3}{12}=2.43\times10^9\ mm^4$$

$$\beta_t=\frac{C}{2I_S}=\frac{2.185*10^9}{2*2.43*10^9}=0.4495$$

-Exterior C.S coefficient% $=100-10\beta t+12\,\beta t\,(\cancelto{0}{\alpha_{f1}}\frac{\ell 2}{\ell 1})\times(1-\frac{\ell 2}{\ell 1})$

$=100-10\times0.4495$

-Exterior C.S coefficient% = 95.505% = 0.95505

Negative moment at column strip = 0.95505 × 90.75 = 86.67 kN.m

Negative moment at middle strip = 90.75 − 86.67 = 4.08 kN.m ■

You can find –ExteriorC.S by interpolation:

$$\frac{\ell_2}{\ell_1}=\frac{5}{6}=0.833$$

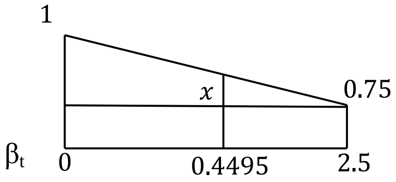
$\beta_t = 0.4495$

$$\frac{1-0.75}{2.5-0}=\frac{x}{2.5-0.4495}$$

X = 0.20502

-Exterior C.S = 0.20502 + 0.75

-Exterior C.S = 0.95505



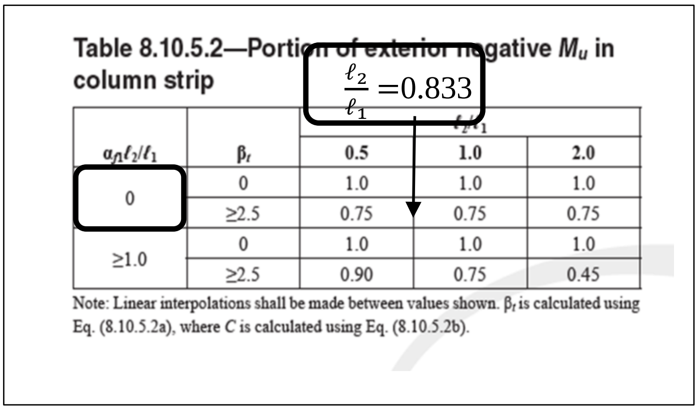


Table 8.10.5.2—Portion of exterior negative M_u in column strip

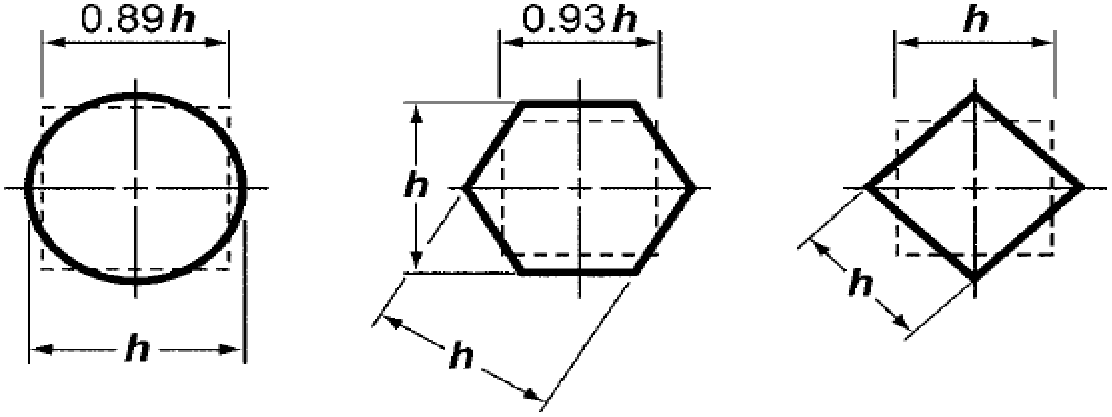
$\frac{\ell_2}{\ell_1}$ =0.833

$\alpha_{f1}\ell_2/\ell_1$	β_t	ℓ_2/ℓ_1		
		0.5	**1.0**	**2.0**
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

Effect of Circular and Capital Columns on Total Static Moment

- ACI states that circular or regular polygon shaped supports shall be treated as square supports having the same area, in calculating clear span ℓ_n.



- For flat slabs, particularly with **column capitals** the clear span ℓ_n computed from using equivalent square supports shall be compared with equation below.

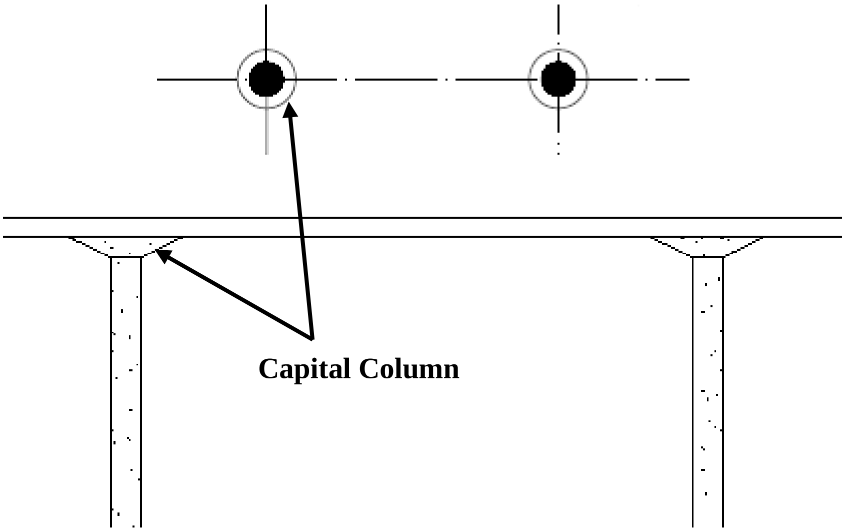
$$M_o = \frac{Wu\,\ell_2\ell_1^2}{8} \times \left(1 - \frac{2c}{3\ell_1}\right)^2 \quad \ldots\ldots\ldots 1$$

$$M_o = \frac{Wu\ell_2\ell_n^2}{8} \quad \ldots\ldots\ldots 2$$

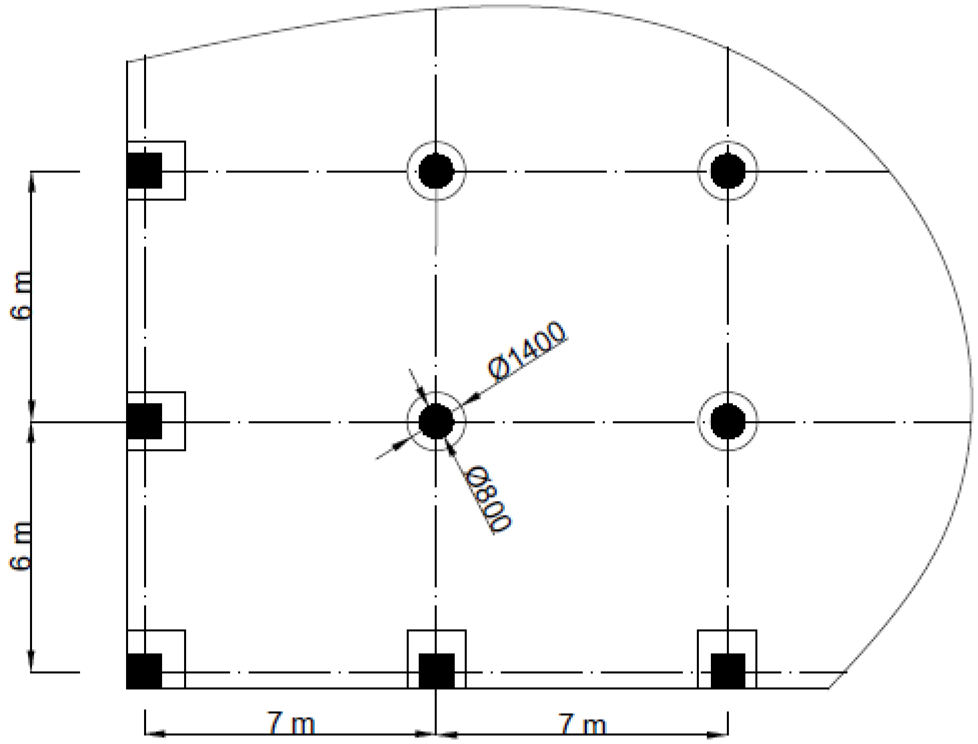
Choose **larger** value of M_o from these two equations.

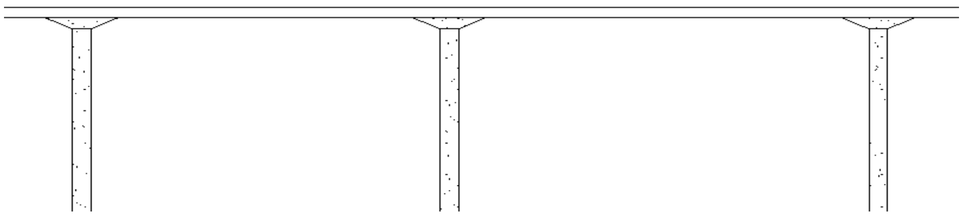
Where **C** is the **larger diameter** of **column** or **capital column** if exist.

- In some cases the latter value is larger and should be used.



Example 9: Compute the total factored static moment in the long and short direction for longitudinal and transverse interior frames for interior panel in flat slab, capital column diameter 1.4 m, circular column diameter 0.8 m and square column (0.8 x 0.8) m. Wu=15 kN/m^2





Side view for the slab

Solution:

For long direction

M$_o$ = $\frac{Wu\,\ell_2\ell_1^2}{8} * (1-\frac{2c}{3\ell_1})^2$ or M$_o$ = $\frac{Wu\ell_2\ell_n^2}{8}$ **choose larger value**

$\ell_1 = 7,\ \ell_2 = 6,\ \ell_n = 7 - 0.89 \times 1.4 = 5.754$ m $> 0.65\ell_1$

M$_o$ = $\frac{15*6*7^2}{8} * (1-\frac{2*1.4}{3*7})^2$ =414 kN.m or M$_o$= $\frac{15*6*5.754^2}{8}$ = 372.4 kN.m <u>**use**</u> **414 kN.m**

For short direction

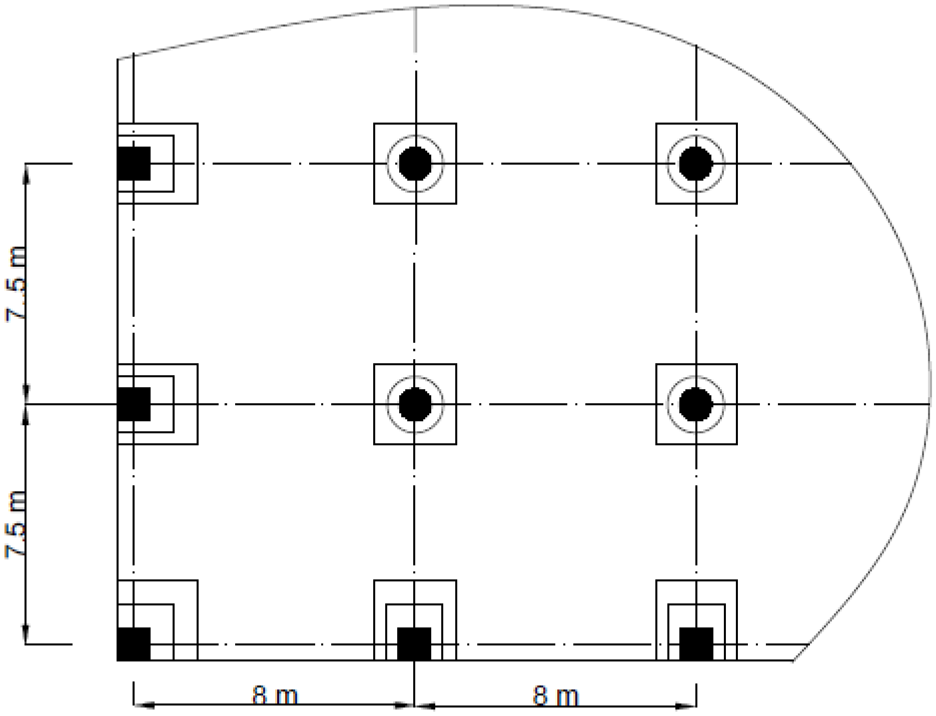
M$_o$ = $\frac{Wu\,\ell_2\ell_1^2}{8} \times (1-\frac{2c}{3\ell_1})^2$ or M$_o$= $\frac{Wu\ell_2\ell_n^2}{8}$ **choose larger value**

$\ell_1 = 6,\ \ell_2 = 7,\ \ell_n = 6 - 0.89 \times 1.4 = 4.754$ m $> 0.65\ell_1$

M$_o$ = $\frac{15*7*6^2}{8} \times (1-\frac{2*1.4}{3*6})^2$ =337 kN.m or M$_o$= $\frac{15*7*4.754^2}{8}$ = 296.6 kN.m <u>**use**</u> **337 kN.m** ■

Home Work Problem 1: Find the total factored static moment for exterior panel for longitudinal and transverse interior frames for previous example.

Home Work Problem 2: Compute the total factored static moment in the long and short direction for an interior frame for interior panel in flat slab, Drop panel dimensions (2 x 2)m capital column diameter 1.4 m, circular column diameter 0.8 m and square column (0.8 x 0.8) m. W_u=15 kN/m^2



Hint for H.W problem 2: A drop panel has **no** effect in compute **clear span (ℓn)**.

ℓ_n for long direction for interior panel $=8 - 0.89 \times 1.4 = 6.754$ m